Some Observations on
Equation-Based Rate Control*

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Problem

Equation-based Rate Control – an approach to do rate control in the Internet

Let $\bar{p}$ be long-run loss ratio; $\hat{p}_n$ be an estimator of $\bar{p}$

Set the send rate $x(t)$ as:

$$x(T_n) = f(\hat{p}_n)$$

at some time points $\{T_n\}$;

else, $x(t) = X(T_n)$, $T_n \leq t < T_{n+1}$;

$f : [0, 1] \rightarrow \mathbb{R}^+$ is a loss-throughput function
A Typical Function $f$

**PFTK formula** ([ToN, 8(2), 2000]):

$$f(p) = \frac{1}{\tau ap^{1/2} + \rho bp^{3/2} + \rho cp^{5/2}} \text{ (pckts/sec)}$$

where $\tau$ and $\rho$ are round-trip time and TCP retransmission timeout, respectively, and $a, b, c$ positive-valued constants.
Problem (cont’d)

Q. Does it hold

\[ E[x(t)] \leq f(\bar{p}) \]

If yes, we say the control is conservative (resp. non-conservative)

Obs. in practice, the expected round-trip time is also estimated; we do not consider this in our study.
Why is the Problem Relevant?

- Many such rate controls are proposed.

- We believe the problem that we pose is not understood for those controls.

- The question is of importance for safe deployment of such rate controls (fair coexistence with TCP – TCP-friendliness).

- There is a lack of analytical studies of such rate controls; the controls are often claimed to be TCP-friendly by over-simplistic fixed-point reasoning.
Outline

We show two major points:

1. Sufficient conditions that ensure conservative control.

2. Identify a cause of empirically observed overly conservative control as loss rate gets high.
Two Specific Assumptions

(A1) $1/\hat{p}_n$ is unbiased estimator of $1/\bar{p}$

(A2) $\{T_n\}$ are loss-event instants

Both assumptions motivated by TFRC (Floyd, Handley, Padhye, and Widmer, 2000)

Obs.[(A1)] Call $\theta_n$ the amount of data sent in $[T_n, T_{n+1})$. Let $\hat{\theta}_n$ be moving-average of $\{\theta_n\}$, i.e.

$$\hat{\theta}_n = \sum_{l=1}^{L} w_l \theta_{n-1}$$

Let $\tilde{p}_n := 1/\hat{\theta}_n$.

By observing $\bar{p} = 1/\mathbb{E}[\theta_n]$, one verifies (A1)

$$\mathbb{E}[1/\tilde{p}_n] = \mathbb{E}[\hat{\theta}_n] = 1/\bar{p}$$

$\tilde{p}_n$ is biased estimator of $\bar{p}$; $\mathbb{E}[\tilde{p}_n] \geq \bar{p}$
A Preliminary Observation

Indeed

$$\mathbb{E}[x(T_n)] = \mathbb{E}[f(\hat{p}_n)]$$

Under (A1), if $f(p)$ is concave w.r.t. $1/p$, we have

$$\mathbb{E}[x(T_n)] \leq f(\bar{p}) \quad (1)$$

But, (1) does not imply $\mathbb{E}[x(t)] \leq f(\bar{p})$!

- The expectation in (1) is Palm expectation (event-average) w.r.t. rate updating events.

- (1) would imply $\mathbb{E}[x(t)] \leq f(\bar{p})$ if $\{T_n\}$ would not depend on the rate process.
Throughput

By Palm inversion formula:

$$\mathbb{E}[x(t)] = \frac{\mathbb{E}[\theta_n]}{\mathbb{E}[S_n]}$$

where $S_n := T_{n+1} - T_n$.

Equivalently,

$$\mathbb{E}[x(t)] = \mathbb{E}[X_n] + \frac{\text{Cov}[X_n, S_n]}{\mathbb{E}[S_n]}$$

where $X_n := x(T_n)$.

Thus, if $\text{Cov}[X_n, S_n] \leq 0$:

$$\mathbb{E}[x(T_n)] \leq f(\bar{p}) \Rightarrow E[x(t)] \leq f(\bar{p})$$
Our First Point
(Sufficient Conditions for a Conservative Control)

Let $\sigma(x) := \mathbb{E}[S_n | X_n = x]$. 

If

(C1) $f(p)$ is concave with $1/p$

(C2) $\sigma(x)$ is non-increasing with $x$

then

$\mathbb{E}[x(t)] \leq f(\bar{p})$

i.e., the control is conservative.
Discussion

- **(C1)** $f(p)$ is concave with $1/p$
  True for some simple formulas (e.g. square-root); not true for PFTK when $p$ is large

- **(C2)** $\sigma(x)$ is non-increasing with $x$
  May not be true for some slowly evolving congestion process
Our Second Point
(Overly Conservative Control for large $\bar{p}$)

It was observed in several empirical studies that TFRC throughput goes to zero as $\bar{p}$ increases.

Q. Why?
Our Second Point (cont’d)

Consider PFTK formula with Bernoulli \( (p) \) loss events \( \sigma(x) = \frac{1}{px} \).

\[
\mathbb{E}[x(t)] = \mathbb{E}\left[ \frac{1}{X_p} \right] \\
= \mathbb{E}\left[ \frac{1}{f(\hat{p}n)} \right] \\
= \frac{1}{\tau a \mathbb{E}[\hat{\theta}_n^{-1/2}] + \rho b \mathbb{E}[\hat{\theta}_n^{-3/2}] + \rho c \mathbb{E}[\hat{\theta}_n^{-5/2}]} \\
\leq \frac{1}{\tau a p^{-1/2} + \rho b p^{-3/2} + \rho c p^{-5/2}}
\]

Obs.

1. \( \hat{\theta}_n^{-1/2} \), \( \hat{\theta}_n^{-3/2} \) and \( \hat{\theta}_n^{-5/2} \) are all convex

2. \( \hat{\theta}_n^{-3/2} \) and \( \hat{\theta}_n^{-5/2} \) are very steep for small \( \hat{\theta}_n \) (large \( \bar{p} \))

This drives the throughput down.
A Numerical Example

\[ f(p) \text{ versus } 1/p \]

Obs. with the square-root formula the phenomena does not exist; with PFTK, yes.
Discussion

In general, deviation of the throughput from $f(\bar{p})$ is due to randomness of the estimator $\hat{p}_n$ and combination of:

1. updating the rate at the loss-event instants (event/time-average)

2. non-linearity of $f$ (convexity/concavity)
Two Non-Conservative Controls

Suppose \( \{S_n\} \) is independent of \( \{x(t)\} \) (e.g. \( \{S_n\} \) i.i.d.).

Then, \( \mathbb{E}[x(t)] = \mathbb{E}[x(T_n)] \)

For both:

1. \( \hat{p}_n \) is unbiased estimator of \( \bar{p} \);
   \( f(p) \) convex w.r.t. \( p \)

2. \( 1/\hat{p}_n \) is unbiased estimator of \( 1/\bar{p} \);
   \( f(p) \) convex w.r.t. \( 1/p \)

\[ \mathbb{E}[x(t)] \geq f(\bar{p}) \]

where equality holds only in degenerate case \( \hat{p}_n \) fixed to \( \bar{p} \).
Conclusion

We have seen:

1. Sufficient conditions under which a TFRC-like equation-based rate control is conservative.

2. A cause of overly conservative TFRC-like control for non-small $\bar{p}$.

Engineering implication:

Be careful with event/time-averages, and nonlinearities of the control!

Not shown in the slides, but present in the paper: