ÉCOLE POLYTECHNIQUE
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Some Observations on Equation-Based Rate Control*

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Problem

Equation-based Rate Control – an approach to do rate control in the Internet

Let $\bar{p}$ be long-run loss ratio; $\widehat{p}_n$ be an estimator of $\bar{p}$

Set the send rate $x(t)$ as:

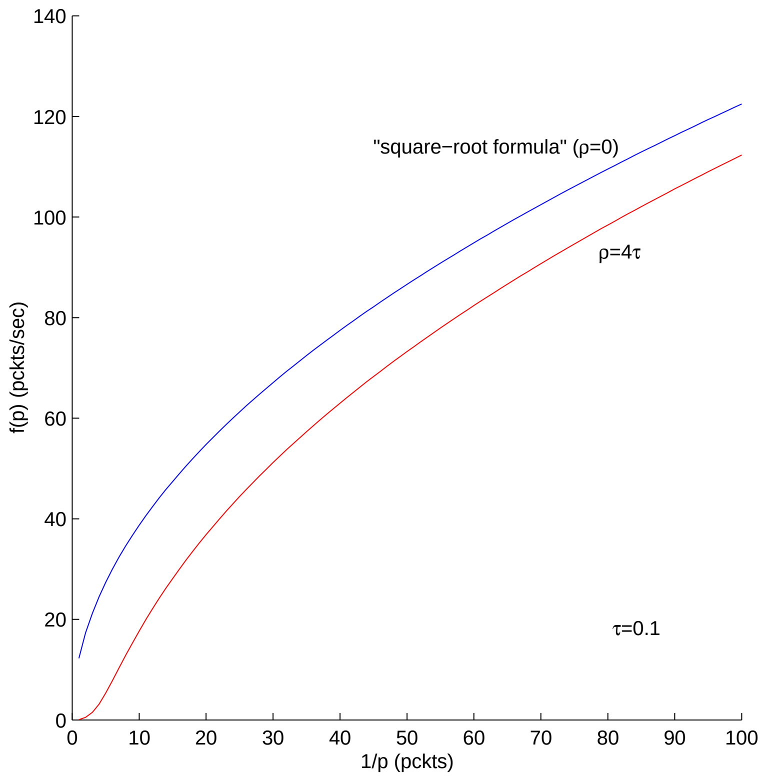
$$x(T_n) = f(\widehat{p}_n)$$

at some time points $\{T_n\}$;

else, $x(t) = X(T_n)$, $T_n \leq t < T_{n+1}$;

$f : [0, 1] \to \mathbb{R}^+$ is a loss-throughput function

A Typical Function f



PFTK formula (ToN, 8(2), 2000):

$$f(p) = \frac{1}{\tau a p^{1/2} + \rho b p^{3/2} + \rho c p^{5/2}} \text{ (pcks/sec)}$$

where τ and ρ are round-trip time and TCP retransmission timeout, respectively, and a, b, c positive-valued constants.

Problem (cont'd)

Q. Does it hold

$$\mathbb{E}[x(t)] \leq f(\bar{p}) \text{ ?}$$

If yes, we say the control is conservative (resp. non-conservative)

Obs. in practice, the expected round-trip time is also estimated; we do not consider this in our study.

Why is the Problem Relevant?

- Many such rate controls are proposed.
- We believe the problem that we pose is not understood for those controls.
- The question is of importance for safe deployment of such rate controls (fair coexistence with TCP – TCP-friendliness).
- There is a lack of analytical studies of such rate controls; the controls are often claimed to be TCP-friendly by over-simplistic fixed-point reasoning.

Outline

We show two major points:

1. Sufficient conditions that ensure conservative control.

2. Identify a cause of empirically observed overly conservative control as loss rate gets high.

Two Specific Assumptions

(A1) $1/\widehat{p}_n$ is unbiased estimator of $1/\bar{p}$

(A2) $\{T_n\}$ are loss-event instants

Both assumptions motivated by TFRC (Floyd, Handley, Padhye, and Widmer, 2000)

Obs.[(A1)] Call θ_n the amount of data sent in $[T_n, T_{n+1})$. Let $\widehat{\theta}_n$ be moving-average of $\{\theta_n\}$, i.e.

$$\widehat{\theta}_n = \sum_{l=1}^{L} w_l \theta_{n-1}$$

Let $\widehat{p}_n := 1/\widehat{\theta}_n$.

By observing $\bar{p} = 1/\mathbb{E}[\theta_n]$, one verifies (A1)

$$\mathbb{E}[1/\widehat{p}_n] = \mathbb{E}[\widehat{\theta}_n] = 1/\bar{p}$$

$\widehat{p}_n$ is biased estimator of $\bar{p}$; $\mathbb{E}[\widehat{p}_n] \geq \bar{p}$

A Preliminary Observation

Indeed

$$\mathbb{E}[x(T_n)] = \mathbb{E}[f(\widehat{p}_n)]$$

Under (A1), if $f(p)$ is concave w.r.t. $1/p$, we have

$$\mathbb{E}[x(T_n)] \leq f(\bar{p}) \tag{1}$$

But, (1) does not imply $\mathbb{E}[x(t)] \leq f(\bar{p})$!

- The expectation in (1) is Palm expectation (event-average) w.r.t. rate updating events.

- (1) would imply $\mathbb{E}[x(t)] \leq f(\bar{p})$ if $\{T_n\}$ would not depend on the rate process.

Throughput

By Palm inversion formula:

$$\mathbb{E}[x(t)] = \frac{\mathbb{E}[\theta_n]}{\mathbb{E}[S_n]}$$

where $S_n := T_{n+1} - T_n$.

Equivalently,

$$\mathbb{E}[x(t)] = \mathbb{E}[X_n] + \frac{\text{Cov}[X_n, S_n]}{\mathbb{E}[S_n]}$$

where $X_n := x(T_n)$.

Thus, if $\text{Cov}[X_n, S_n] \leq 0$:

$$\mathbb{E}[x(T_n)] \leq f(\bar{p}) \ \Rightarrow E[x(t)] \leq f(\bar{p})$$

Our First Point
(Sufficient Conditions for a Conservative Control)

Let $\sigma(x) := \mathbb{E}[S_n | X_n = x]$.

If

(C1) $f(p)$ is concave with $1/p$

(C2) $\sigma(x)$ is non-increasing with x

then

$$\mathbb{E}[x(t)] \leq f(\bar{p})$$

i.e., the control is conservative.

Discussion

- **(C1)** $f(p)$ is concave with $1/p$
  True for some simple formulas (e.g. square-root); not true for PFTK when p is large

- **(C2)** $\sigma(x)$ is non-increasing with x
  May not be true for some slowly evolving congestion process

Our Second Point
(Overly Conservative Control for large $\bar{p}$)

It was observed in several empirical studies that TFRC throughput goes to zero as $\bar{p}$ increases.

Q. Why?

Our Second Point (cont'd)

Consider PFTK formula with Bernoulli (p) loss events ($\sigma(x) = \frac{1}{px}$).

$$\begin{aligned}
\mathbb{E}[x(t)] &= \frac{1}{\mathbb{E}[\frac{1}{X_n}]} \\
&= \frac{1}{\mathbb{E}[\frac{1}{f(\widehat{p}_n)}]} \\
&= \frac{1}{\tau a \mathbb{E}[\widehat{\theta}_n^{-\frac{1}{2}}] + \rho b \mathbb{E}[\widehat{\theta}_n^{-\frac{3}{2}}] + \rho c \mathbb{E}[\widehat{\theta}_n^{-\frac{5}{2}}]} \\
&\leq \frac{1}{\tau a \bar{p}^{1/2} + \rho b \bar{p}^{3/2} + \rho c \bar{p}^{5/2}}
\end{aligned}$$

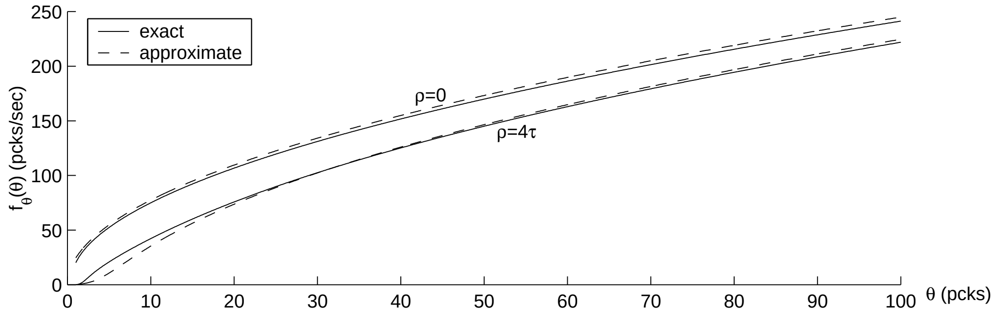
Obs.

1. $\widehat{\theta}_n^{-1/2}$, $\widehat{\theta}_n^{-3/2}$ and $\widehat{\theta}_n^{-5/2}$ are all convex
2. $\widehat{\theta}_n^{-3/2}$ and $\widehat{\theta}_n^{-5/2}$ are very steep for small $\widehat{\theta}_n$ (large $\bar{p}$)

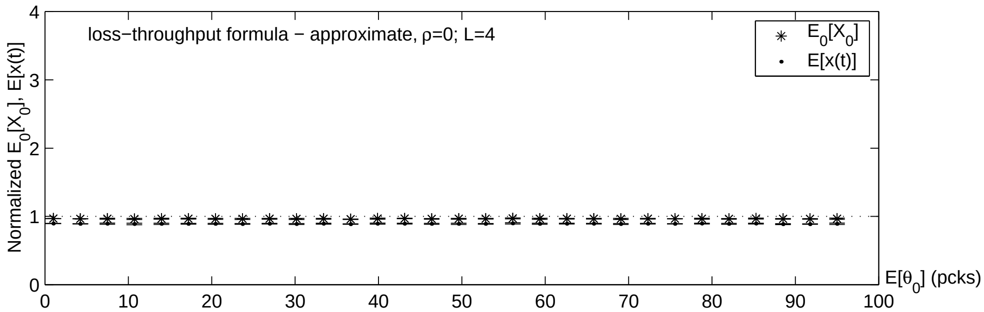
This drives the throughput down.

A Numerical Example

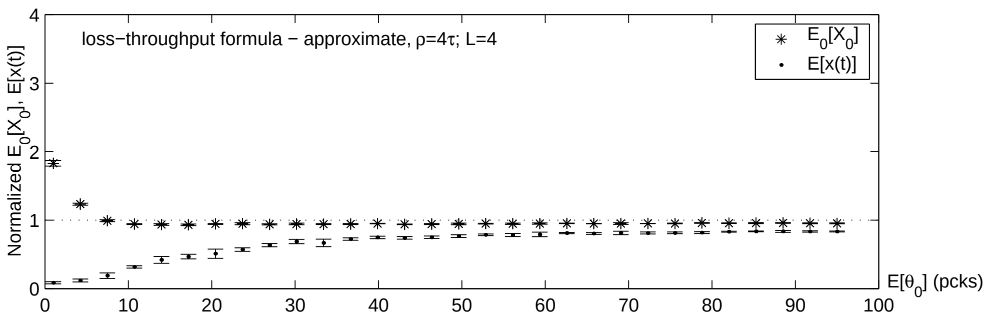
$f(p)$ versus $1/p$



Square-root



PFTK



Obs. with the square-root formula the phenomena does not exist; with PFTK, yes.

Discussion

In general, deviation of the throughput from $f(\bar{p})$ is due to randomness of the estimator $\widehat{p}_n$ and combination of:

1. updating the rate at the loss-event instants (event/time-average)

2. non-linearity of f (convexity/concavity)

Two Non-Conservative Controls

Suppose $\{S_n\}$ is independent of $\{x(t)\}$ (e.g. $\{S_n\}$ i.i.d.).

Then, $\mathbb{E}[x(t)] = \mathbb{E}[x(T_n)]$

For both:

1. $\widehat{p}_n$ is unbiased estimator of $\bar{p}$;
   $f(p)$ convex w.r.t. p

2. $1/\widehat{p}_n$ is unbiased estimator of $1/\bar{p}$;
   $f(p)$ convex w.r.t. $1/p$

$$\mathbb{E}[x(t)] \geq f(\bar{p})$$

where equality holds only in degenerate case $\widehat{p}_n$ fixed to $\bar{p}$.

Conclusion

We have seen:

1. Sufficient conditions under which a TFRC-like equation-based rate control is conservative.

2. A cause of overly conservative TFRC-like control for non-small $\bar{p}$.

Engineering implication:

Be careful with event/time-averages, and non-linearities of the control!

Not shown in the slides, but present in the paper:

1. Modeling. Computation of the throughput for hidden Markov chain driven inter loss-event times $\{S_n\}$.