Introduction to Digital Signal Processing

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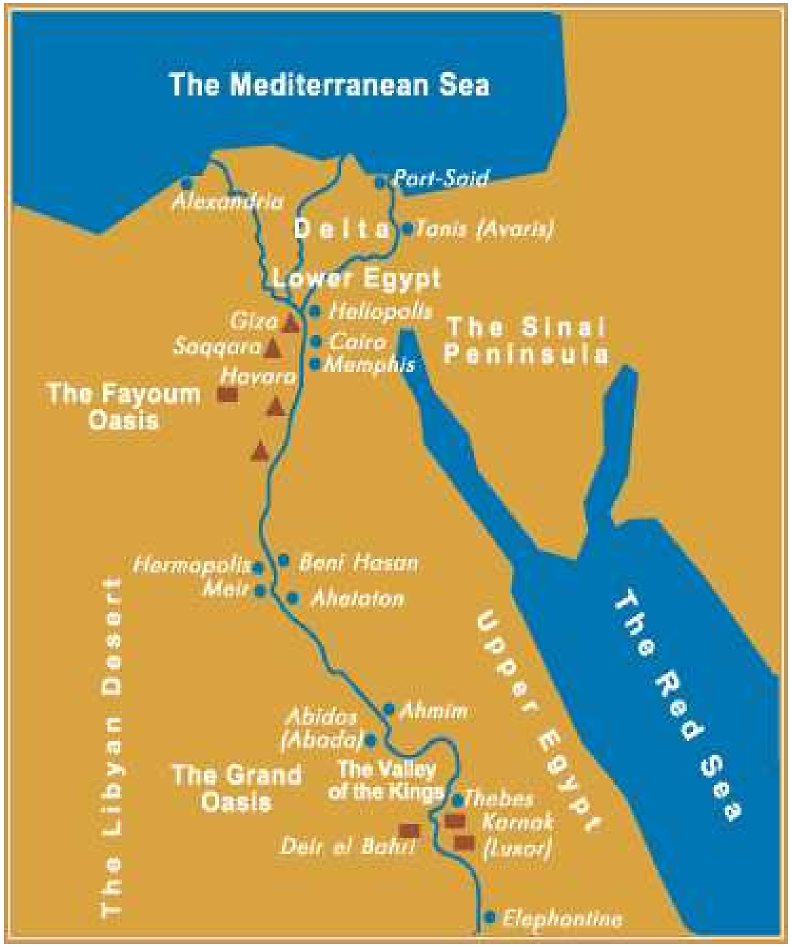
Inside DSP. . .

- **Digital**
  - Brings experimental data & abstract models together
  - Makes math very simple i.e. *implementable*
- **Signal**
  - Measurement of a varying quantity
  - Experimental data (physics, electronics, astronomy, etc.)
- **Processing**
  - Manipulation of the information content
  - Abstract model (math, computer science, etc.)

A Bit of History and Philosophy

Egypt, 2500 BC:

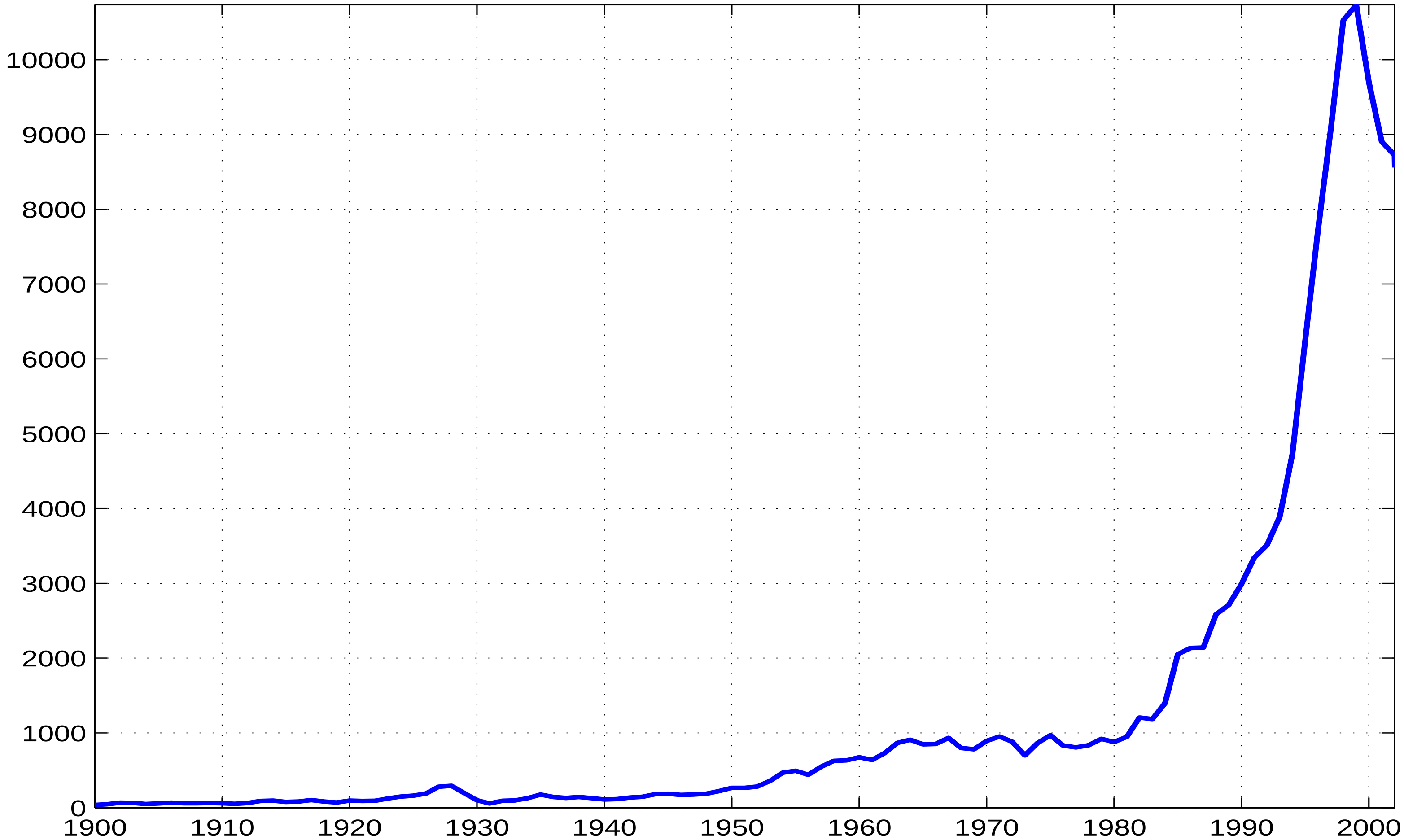


A Bit of History and Philosophy

Egypt, 2500 BC: the Palermo stone.

A Bit of History and Philosophy

USA, 2005 AD: the Dow-Jones Industrial Average



A Bit of History and Philosophy

What do these measurements have in common?

- Life-changing phenomena
- Unpredictable patterns
- Discrete set of observations

= Digital Signal Processing

Is a discrete set of measurement a sufficient representation?

Can we formalize this concept?

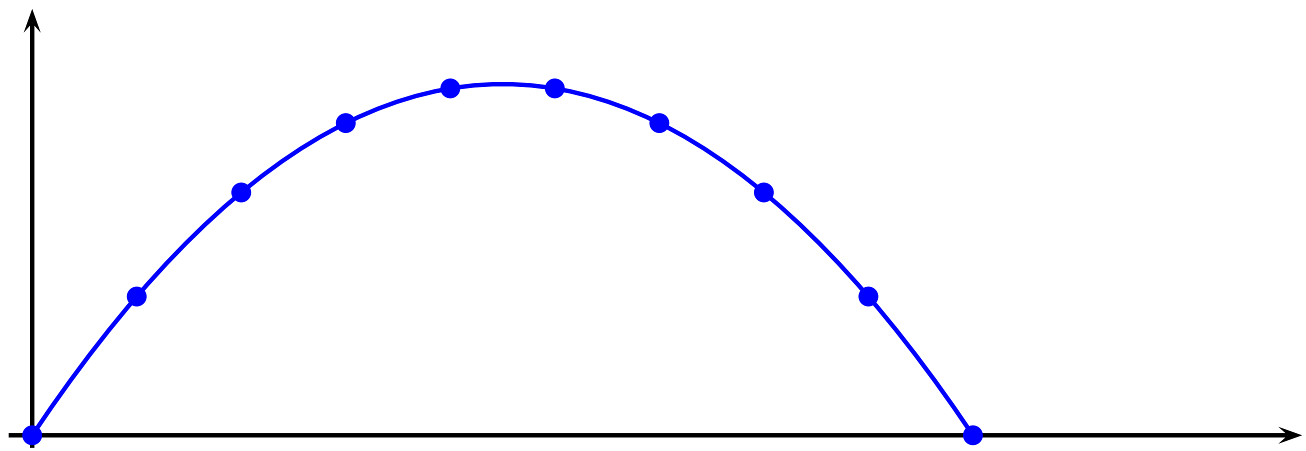
A Bit of History and Philosophy

The Platonic schizophrenia of Western thought.

- Dichotomy between the ideal and the real
- Zeno’s paradoxes
- An odd synergy: calculus and ballistics

A Bit of History and Philosophy

Calculus: a lofty ideal at the service of war.

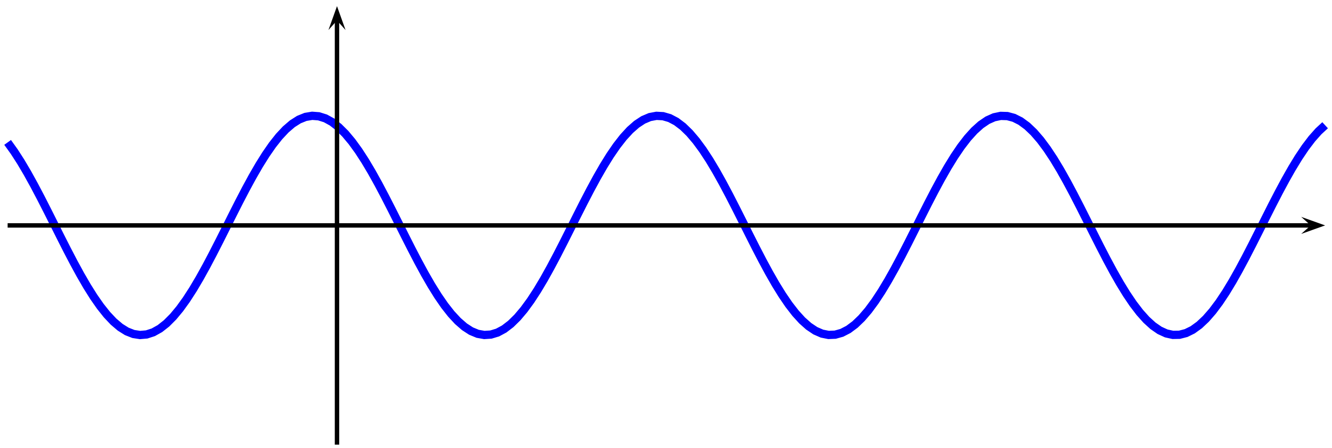


$$\vec{x}(t) = \vec{v}_0 t + (1/2)\vec{g}\,t^2$$

Galileo, 1638

Ideal Signals vs. Real Signals

How does an ideal signal look like? Tuning fork:



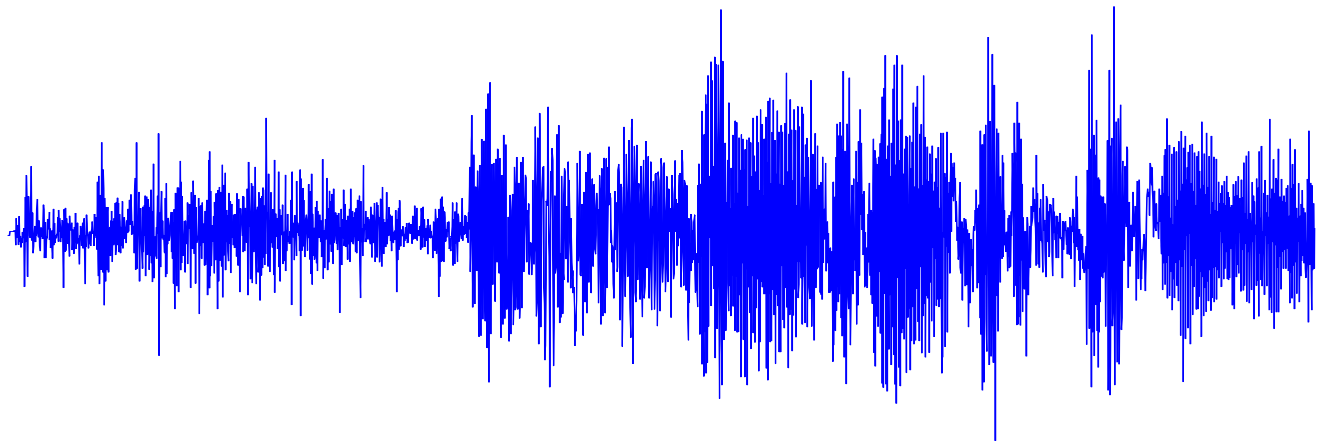
It's a function of a real variable!

$$f(t) = A\sin(2\pi\omega t + \phi)$$

As such, 3 parameters *completely* describe the signal.

Ideal Signals vs. Real Signals

Tuning forks are boring; Bach is not:



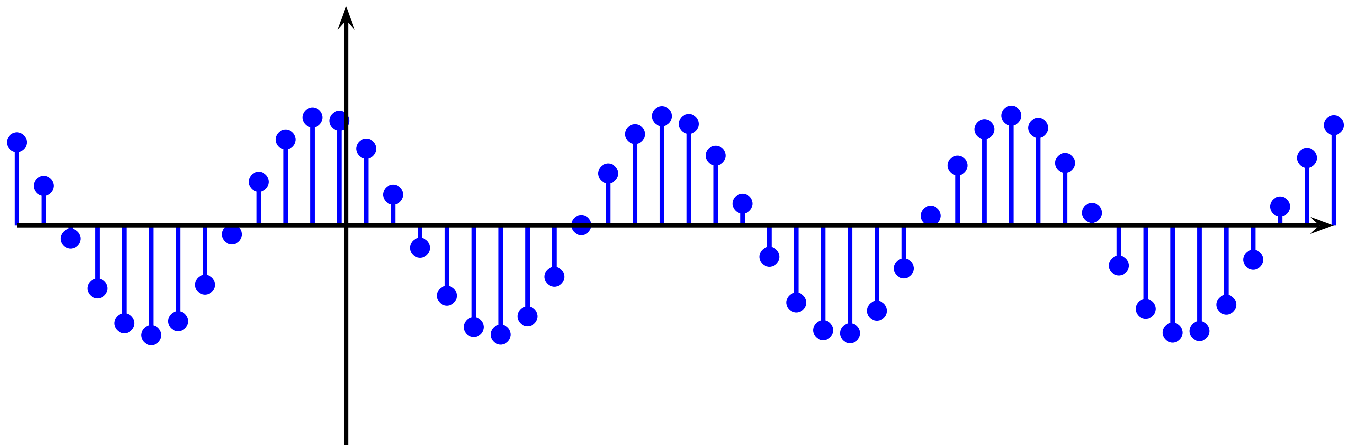
Unfortunately (or fortunately):

$$f(t) = ?$$

How do we deal with real-world signals?

Ideal Signals vs. Real Signals

Sampling: we measure the signal value at regular intervals



$$x[n] = f(nT_s)$$

Can we do this or are we in one of Zeno's paradoxes?
Yes, we can if the signal is "slow enough".

Ideal Signals vs. Real Signals

The Sampling Theorem (Nyquist 1920).
Under appropriate "slowness" conditions for $f(t)$ we have:

$$f(t) = \sum_{n=-\infty}^{\infty} x[n] \frac{\sin(\pi(t - nT_s)/T_s)}{\pi(t - nT_s)/T_s}$$

In a way, the sampling theorem solves one of Zeno's paradoxes: the infinite and the finite have been reconciled.

The sampling theorem is the "revolving door" into the digital world.
We will therefore operate in the digital world only.

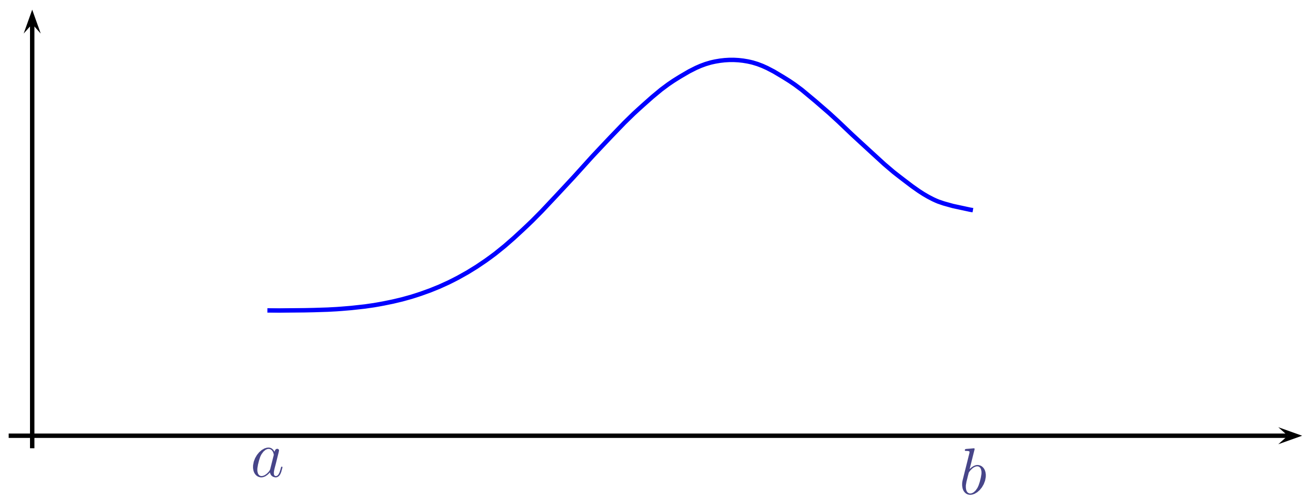
The Digital Revolution

Digital signals make our life simpler:

- Processing:
  - Sequence of numbers: ideal for computations
  - Development easy (general-purpose hardware)
- Storage:
  - Storage is basically media-independent
  - Perfect duplication
  - Digital compression is miraculous
- Communications:
  - Transmission schemes independent of data
  - Error correction techniques make it noise-free

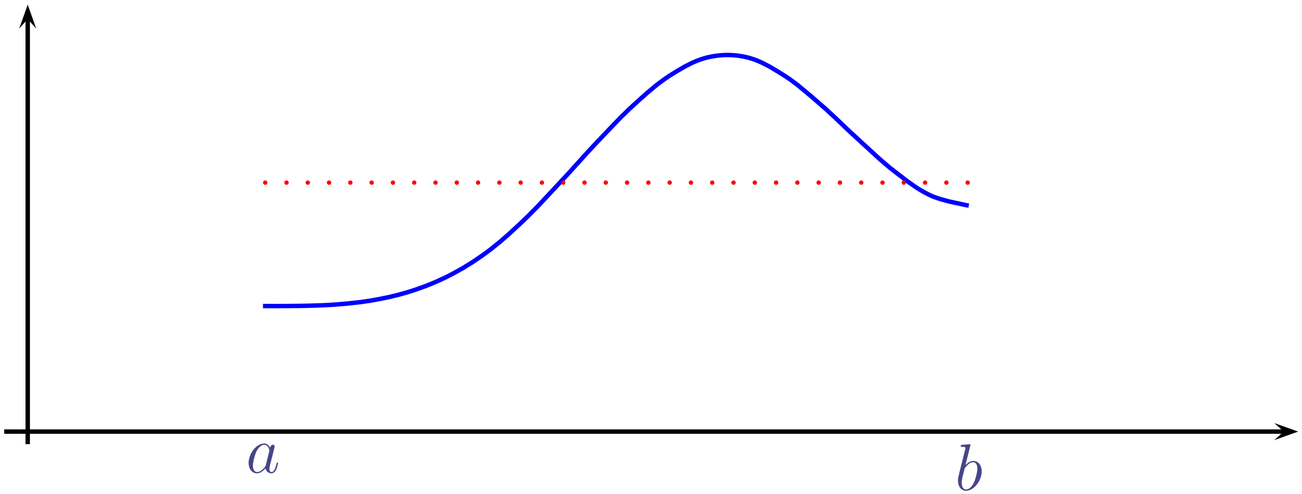
The Digital Revolution: Processing

Computing the average value of a signal.



The Digital Revolution: Processing

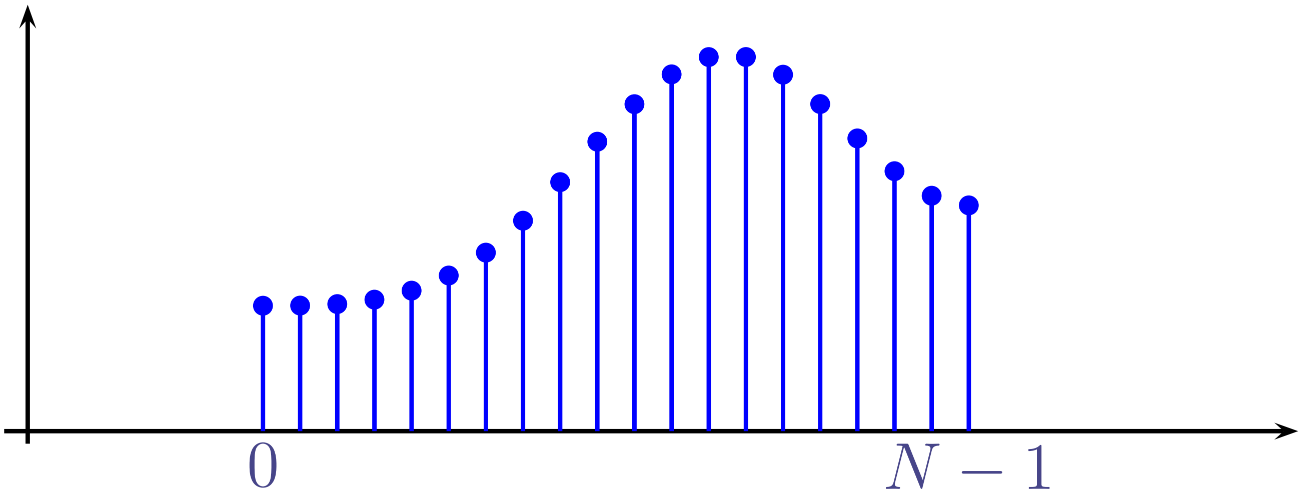
Computing the average value of a signal.



$$\bar{x} = \frac{1}{b-a}\int_a^b f(t)dt$$

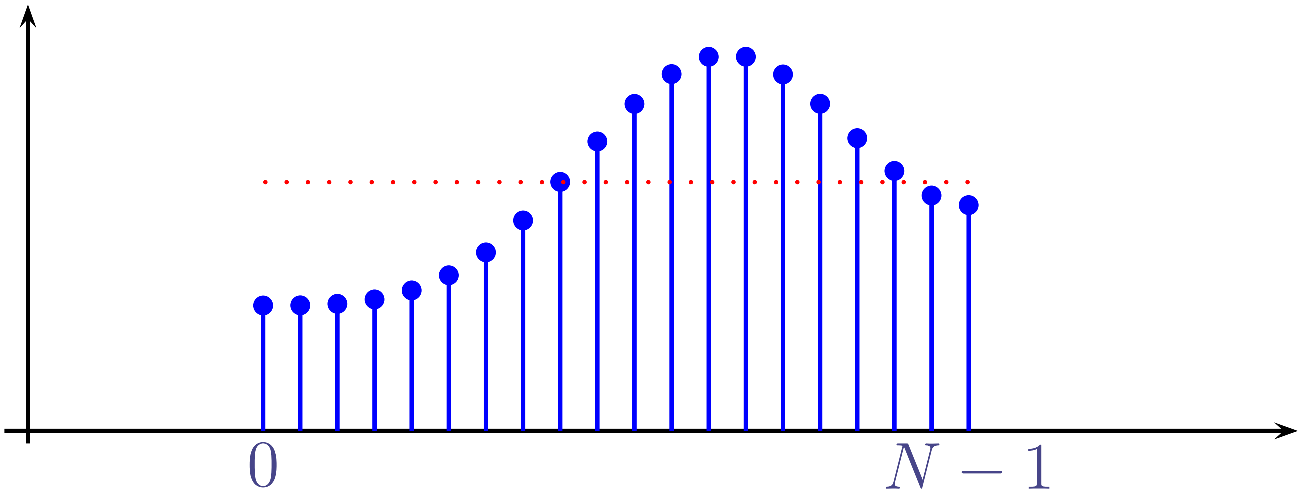
The Digital Revolution: Processing

Computing the average value of a *digital* signal.



The Digital Revolution: Processing

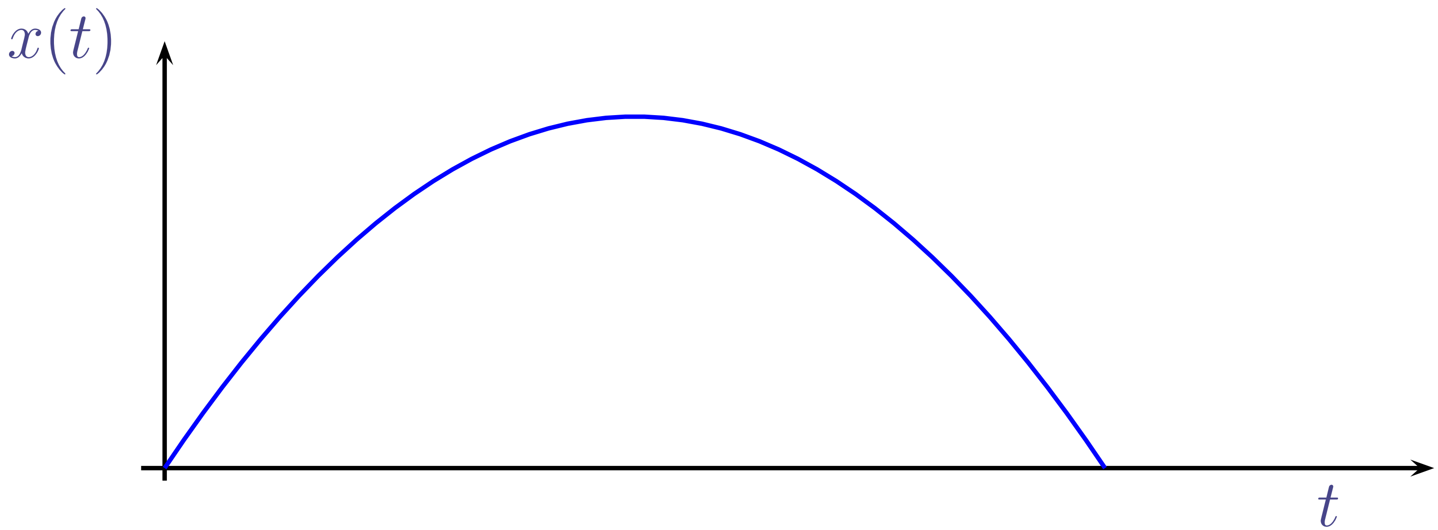
Computing the average value of a *digital* signal.



$$\bar{x} = \frac{1}{N} \sum_{n=0}^{N-1} x[n]$$

The Digital Revolution: Processing

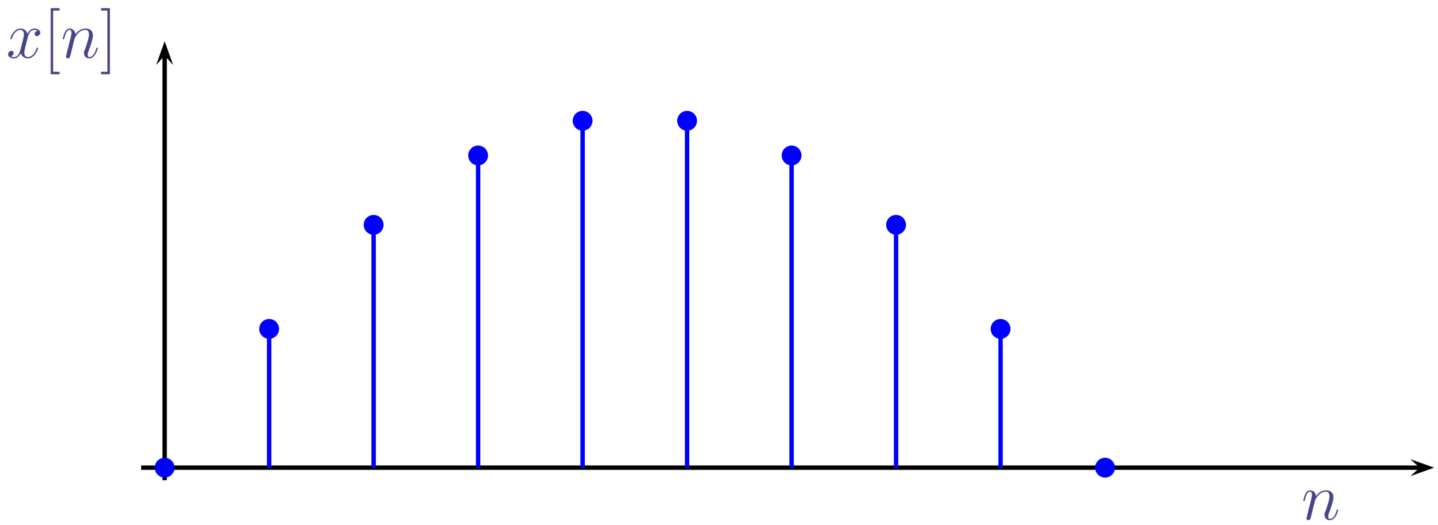
Computing (vertical) speed the "Platonic" way.



$$x(t) = v_0 t - (1/2)gt^2$$
$$v(t) = \dot{x}(t) = v_0 - gt$$

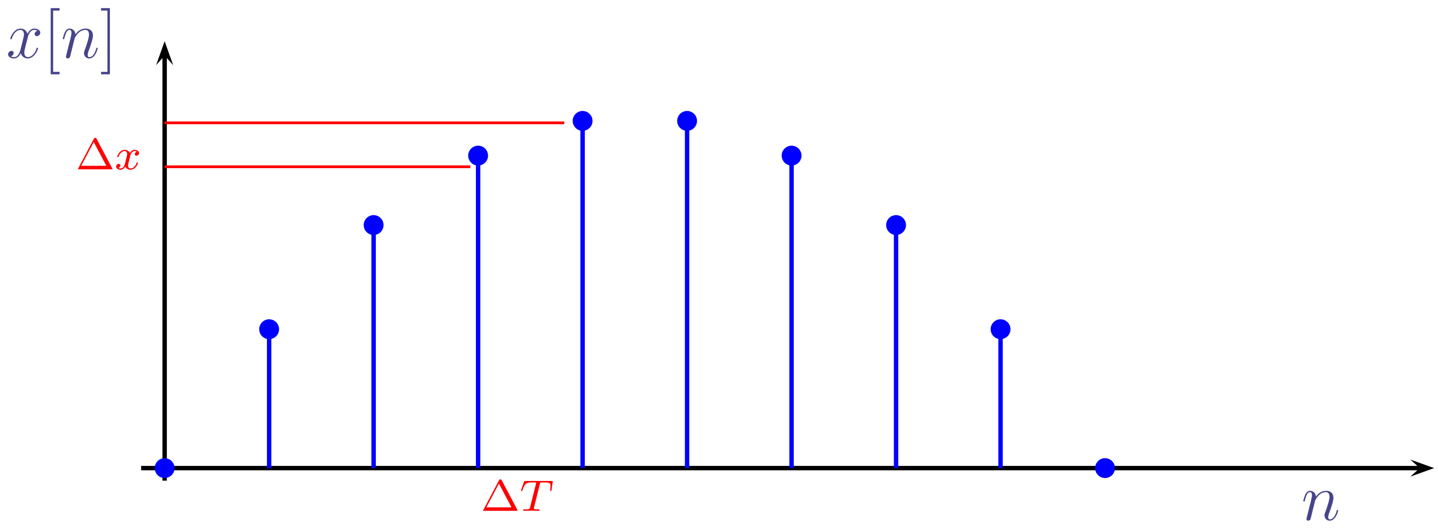
The Digital Revolution: Processing

Computing speed the DSP way.



The Digital Revolution: Processing

Computing speed the DSP way.



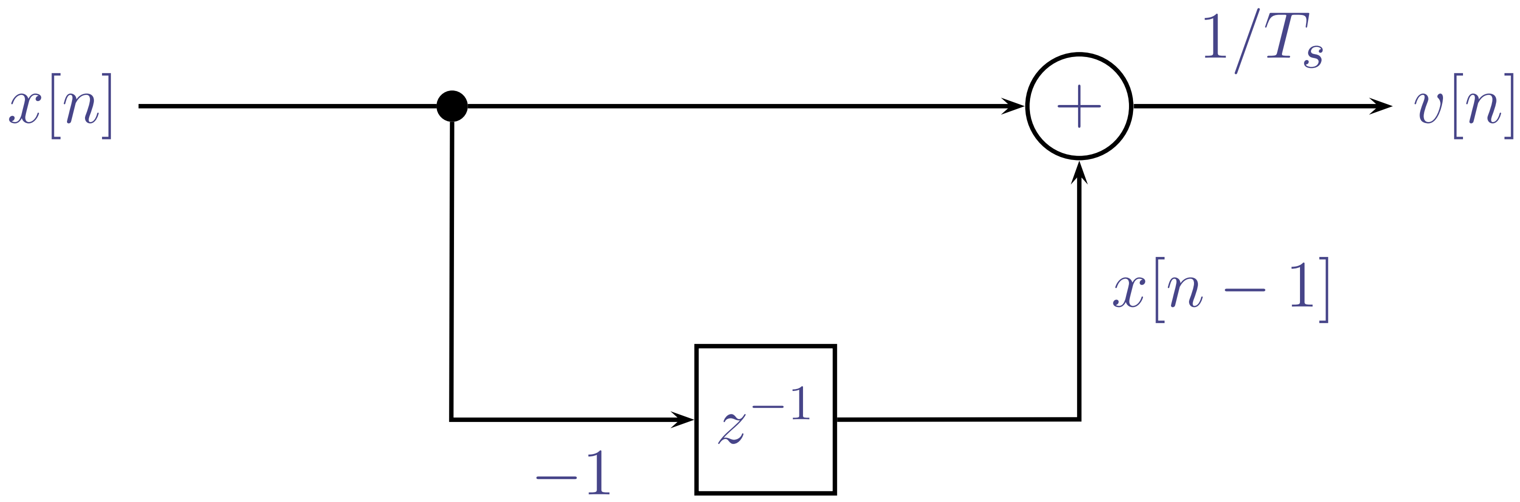
$$v[n] = (x[n] - x[n-1])/T_s$$

The Digital Revolution: Processing

The "Speed Filter":

The Digital Revolution: Processing

Inside the "Speed Filter":



This is a general results: filters' building blocks are just delays, multiplications and additions.

The Digital Revolution: Storage

How do you store a signal?

- In the (not so) old days:
  - Build a physical system (wax cylinders, magnetic tapes, vynil...)
  - Fragile, data dependent
- Nowadays:
  - Quantize the signal values into binary digits
  - Store in any digital memory support
  - Perfect copies

Signal to noise ratio for digital signals:

SNR $\approx$ 6 dB / bit

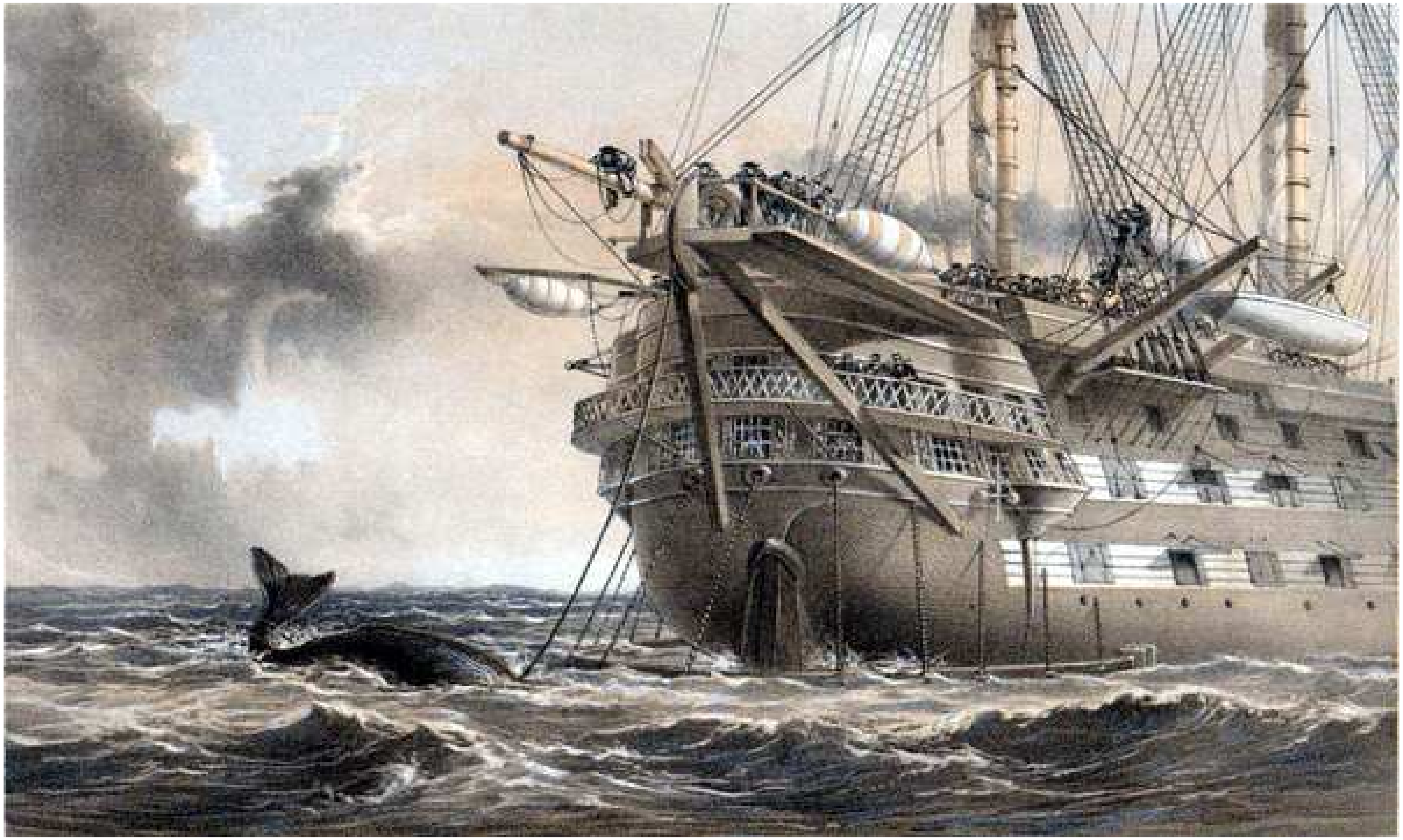
The Digital Revolution: Storage

How do you deal with large amounts of data? Compression!

Signal Type	Default Rate	Compressed Rate
Music	4.32 Mbps CD audio	128 Kbps MP3
Voice	64 Kbps AM radio	4.8 Kbps CELP
Image	20 Mb this image	600 Kb JPEG
Video	170 Mbs PAL video	600-800 Kbs DiVx

The Digital Revolution: Transmission

The Agamemnon, 1858



The Digital Revolution: Transmission

Digital data allows for large throughputs:

- Transoceanic cable:
  - 1866: 8 words per minute ($\approx$5 bps)
  - 1956: AT&T, coax, 48 voice channels ($\approx$3Mbps)
  - 2005: Alcatel Tera10, fiber, 8.4 Tbps (10^{12} bps)

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- Voiceband modems:
  - 1950s: Bell 202, 1200 bps
  - 1990s: V90, 56000bps

DSP Friends and Partners

- Electronics
- Computer science
- Physiology
- Music
- Medicine
- Photography
- And many more...

Conclusions

Digital signal processing is FUN!

It's a fresh new take on what you already studied in theory.

Just turn on a computer and you have a "mad scientist lab" where you can bring everything you know, and nothing ever blows up.