Linear Regression

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1 Model: Linear Regression

What is it?

Linear regression is a model that assumes a linear relationship between inputs and the output.

Why learn about linear regression?

Plenty of reasons: simple, easy to understand, most widely used, easily generalized to non-linear models. Most importantly, you can learn almost all fundamental concepts of ML with regression alone.
Simple linear regression

With only one input dimension, it is simple linear regression.

\[ y_n \approx f(x_n) := \beta_0 + \beta_1 x_{n1} \]

Here, \( \beta_0 \) and \( \beta_1 \) are parameters of the model.

Multiple linear regression

With multiple input dimension, it is multiple linear regression.

\[ y_n \approx f(x_n) := \beta_0 + \beta_1 x_{n1} + \ldots + \beta_D x_{nD} \]
\[ = \bar{x}_n^T \beta \]  

Learning/estimation/fitting

Given data, we would like to find \( \beta = [\beta_0, \beta_1, \ldots, \beta_D] \). This is called learning or estimating the parameters or fitting the model.
Additional Notes

Matrix multiplication

To go any further, one must revise matrix multiplication. Remember that multiplication of $M \times N$ matrix with a $N \times D$ matrix results in an $M \times D$ matrix. Also, two matrices of size $M \times N_1$ and $M \times N_2$ can only be multiplied when $N_1 = N_2$.

$p > n$ Problem

Consider the following simple situation: You have $N = 1$ and you want to fit $y_1 \approx \beta_0 + \beta_1 x_{11}$, i.e. you want to find $\beta_0$ and $\beta_1$ given one pair $(y_1, x_{11})$. Is it possible to find such a line?

This problem is related to something called $p > n$ problem. In our notation, this will be called $D > N$ problem, i.e. the number of parameters exceeds number of data examples.

Similar issues will arise when we use gradient descent or least-squares to fit a linear model. These problems are all solved by using regularization, which we will learn later.