Quality-Aware Routing Metrics for Time-Varying Wireless Mesh Networks

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Abstract—This paper considers the problem of selecting good paths in a wireless mesh network. It is well-known that picking the path with the smallest number of hops between two nodes often leads to poor performance, because such paths tend to use links that could have marginal quality. As a result, quality-aware routing metrics are desired for networks that are built solely from wireless radios. Previous work has developed metrics (such as ETX) that work well when wireless channel conditions are relatively static [1], but typical wireless channels experience variations at many time-scales. For example, channels may have low average packet loss ratios, but with high variability, implying that metrics that use the mean loss ratio will perform poorly. In this paper, we describe two new metrics, called mETX (modified expected number of transmissions) and ENT (Effective Number of Transmissions) that work well under a wide variety of channel conditions. In addition to analyzing and evaluating the performance of these metrics, we provide a unified geometric interpretation for wireless quality-aware routing metrics. Empirical observations of a real-world wireless mesh network suggest that mETX and ENT could achieve a 50% reduction in the average packet loss rate compared to ETX.

Index Terms—Wireless networks, mesh networks, routing protocols, path selection, quality-aware routing, wireless channel modeling, large deviations, effective bandwidth.

I. INTRODUCTION

This paper considers the problem of selecting good paths in networks made up of multiple wireless links, such as wireless mesh networks and wireless sensor networks. By “good paths”, we mean paths that both benefit individual data transfers (in terms of TCP connection throughput, for example), and which lead to high aggregate network capacity.

This problem is considerably harder in wireless networks than in traditional wired networks (where the routing problem is usually solved by running a distributed shortest-path algorithm on a graph) because the notion of a “link” between nodes is not well-defined. The properties of the radio channel between any pair of nodes vary with time, and radio communication range is often unpredictable. The communication quality of a radio channel depends on background noise, obstacles, and channel fading, as well as on other transmissions occurring simultaneously in the network.

Recent research has shown that quality-aware routing (QAR) metrics, such as the Expected Transmission Count (ETX) [1], can improve the throughput of wireless mesh networks by significant amounts compared to traditional shortest-hop-count routing protocols. Although the performance improvement of ETX over the minimum hop-count metric is impressive, it does not cope well with short-term channel variations because it uses the mean loss ratios in making routing decisions. For example, radio channels may have low average packet loss ratios, but with high variability, implying that metrics that use the mean loss ratio will perform poorly because they don’t adapt well to burst loss conditions. The next section presents empirical observations of real-world wireless mesh networks where such conditions occur.

In this paper, we first develop a modified version of ETX, called mETX, that corrects this shortcoming (§IV). Then, we turn our attention to the problem of routing packets in a way that takes higher-layer protocol requirements into account, finding a path that achieves high network capacity while ensuring that the end-to-end packet loss rate visible to higher layers (such as TCP) does not exceed a specified value. We develop a new routing metric, called Effective Number of Transmissions (ENT), for this purpose (§V). We analyze the differences between mETX and ENT from previously proposed routing metrics (see §II), providing a unified geometric framework that combines the mean and standard deviation of channel loss ratios to physically interpret these different metrics and understand their relative benefits and shortcomings.

Both the mETX and the ENT capture the time varying characteristics of a wireless channel in a form that could be directly translated into network and application layer quality constraints. In a sense, they project both the mean and the variance of certain parameters of the physical layer onto the space of networking parameters using tools from large deviations theory and effective bandwidths. This enables us to evaluate the impact of physical layer variability on the network level performance. We show that by taking this “relevant” portion of variability into account, we can achieve a 50% reduction in the average packet loss rate in the Roofnet. Note that our algorithms are by no means universal in modeling and quantifying the variability. In that sense, one should focus on the ideas and the insights rather than the details of the ways we implement the routing algorithms.

To cope with channel variations, modern radios (e.g., 802.11 chipsets) use adaptive modulation schemes, allowing higher layers to set one of several possible bit rates. If frame loss rates at a particular bit rate rise, reducing the bit rate can reduce the observed frame loss ratio and improve throughput. Several bit rate adaptation schemes have been proposed (see [2] for a detailed treatment), and the topic remains an active area of work. We view bit rate selection as being complementary to quality-aware routing, in the sense that once the routing protocol picks the best neighbor to use for a destination using measured cost metrics, the link layer picks the best bit rate (modulation scheme) to use for that neighbor. Thus, the only...
modification required for ETX when bit rate selection is used is to use the Expected Transmission Time (rather than Count) as the metric [3], because a lower bit rate ends up using the channel for a longer period of time.\(^1\) In the same way, the mETX and ENT metrics proposed and analyzed in this paper can be adapted easily for radios that provide bit rate adaptation. For simplicity, however, we present our algorithms assuming a single-rate radio.

II. MOTIVATION AND RELATED WORK

The traditional approach to routing in ad hoc wireless networks is minimum-hop (shortest-path) routing (e.g., [4], [5]). Although simple, minimum-hop routing inherently “quantizes” the state of a link\(^2\) into one of the two states, “up” or “down.” Several researchers have described why minimum-hop routing in wireless networks leads to sub-optimal performance [6], [1]—such routing leads to paths that use longer-range links of marginal quality. QAR metrics counter these performance problems by using observations of frame delivery, signal strength, etc. in picking paths for packets.

We observe that the type of QAR metric to be chosen depends on the physical layer being used. Designing and implementing a physical layer that can fully “hide” the vagaries of the radio channel from higher layers has proven to be difficult for a number of reasons. It requires the physical layer to be able to accurately estimate and adapt several parameters (e.g., transmit power, modulation, error control coding, etc.) to cope with channel conditions that vary rapidly in time. In fact, we know of no current or next-generation radios that propose to employ sophisticated techniques to fully handle channel quality issues at the physical layer, because of implementation complexity and the absence of practically useful codes that can perform well (especially in the non-asymptotic limit of finite packet sizes) across the large range of channel conditions that are observed in practice.

Indeed, practical wireless radios such as the ones based on the various IEEE 802 standards (e.g., 802.11, 802.15, etc.) employ only a simple coding strategy, mostly for error detection. Nodes transmit at one of a discrete set of power levels, and rely on a small number of link-layer packet retransmissions to overcome errors. For instance, many 802.11 networks vary the modulation (and hence the rate) based on current error conditions, slowing the symbol transmission rate when the error rate is high, and use link-layer retransmissions to attempt to recover from frame losses. All other packet losses are visible to higher layers, where they may be recovered using end-to-end mechanisms (such as TCP retransmissions or packet-level forward error correction implemented by applications). Our work focuses on developing QAR metrics over radio networks comprised of radios similar to 802.11.

A. Related Work

Our QAR metrics are mainly inspired by ETX, a metric proposed by De Couto et al. [1] for 802.11-based radios

\(^1\)Abusing terminology slightly, we use the term ETX to refer to the Expected Transmission Time metric in this paper.

\(^2\)We use the term “link” to refer to the communication channel between a pair of nodes.

employing link-layer retransmissions to recover from frame losses. The ETX of a radio link is the average number of \{data frame, ACK frame\} transmissions necessary to transfer a packet successfully over a wireless link. In ETX, each node estimates the frame loss ratio \(p_f\) to each of its neighbors over a recent time window, and obtains an estimate \(p_e\) of the reverse direction from its neighbor (these loss estimates are obtained using broadcast packets that are not retransmitted at the link layer). The node then estimates the expected transmission (ETX) count to a neighbor as \(\frac{1}{(1-p_f)(1-p_e)}\), and picks the path that has the smallest ETX value from a set of choices.

Yarvis et al. [6] propose a QAR metric that estimates the per-link frame delivery ratios and uses the end-to-end path loss probability as the cost metric of a path. This metric does not account for the total bandwidth consumed, because it will prefer two links of low frame loss ratio over a single link with higher loss-rate; when link-layer retransmissions are used, the single higher-loss link may be able to deliver the packet without as many total transmissions as the two-hop path (ETX is motivated by this observation).

Adya et al. [7] propose a delay-based QAR metric. This metric uses the measured average round trip time (RTT) seen by unicast probes between neighboring nodes. Draves et al. [8] present an extensive experimental comparison of ETX and the RTT metrics. They find the RTT metric to be load-sensitive and to perform poorly in some cases due to self-interference. They also find that pure hop count can outperform these QAR metrics under the presence of mobility and high channel variability since it reacts more quickly to fast topology changes.

B. Channel Variability

The number of transmissions of a packet on a radio link is an appealing cost metric because minimizing the total number of transmissions maximizes the overall throughput. Moreover, this metric minimizes the transmission energy consumed in
transferring a packet along a path in a network when the nodes transmit at a constant power level.

Although the experimental results in [1], [8] show that ETX performs better than traditional shortest path routing under static network conditions, it may perform poorly under highly variable channel conditions and burst-loss situations because ETX considers only the average channel behavior. In particular, the routing protocol measures the channel state using a set of probe packets sent once every second, averaging the loss ratio over an interval of about 10 seconds. The reciprocal of this estimate is assigned as the ETX of the link. In this procedure, the number of transmissions is implicitly assumed to be a geometric random variable; if successive packets are lost independently with probability equal to the average packet error rate of the channel, the assumption is accurate. Packet losses generally occur in bursts, however, and the packet loss probability is usually not constant.

For instance, Fig. 1 illustrates the packet delivery ratios taken from four distinct links in the Roofnet wireless mesh network [9]. Each node in the network has an 802.11b wireless card and an antenna. The transmission rate is set to a constant 11 Mbps. The delivery ratios were obtained by sending a sequence of 1500-byte broadcast packets, with the receiver keeping track of which packets were received successfully. The successful receipt or loss of a packet defines a binary random variable; each sample delivery ratio in the graphs is the average of a window of 40 successive binary random variables. The window advances by 1 for each reported sample.

Each of these four links has an ETX of approximately 2 during the testing period. Therefore, if ETX is taken as the metric for quality, these four links are identical. On the other hand, the sample variances of the delivery ratios are quite different for these links. These wireless links behave similarly in the long-term even though their short-term behavior quite different. Indeed, the sample coefficient of variation for the binary packet error sequences are 7.92, 2.16, 1.20 and 0.61.

One may ask whether it is possible to update the ETX measurements and accordingly update the optimum path more and more frequently until the “remaining” variability between updates is somewhat insignificant. Unfortunately, the update procedure involves significant amount of overhead in the network, and if repeated frequently it causes inefficient use of resources, extra interference and even instability of the routing algorithm. Therefore, the time-scale over which path-selection decisions are made is typically no less than tens or hundreds of packets; i.e., once a path between two nodes has been selected, it is unlikely to be changed from packet to packet. As shown in Fig. 1, there may be a huge channel variability over the time-scale of the transmission of a that many packets.

These empirical mesh network measurements lend further credence to the conclusions of several previous measurement studies, which have shown that such variable behavior is common in all wireless communications. Willig et al. present the analysis of packet traces over 2.4 GHz 802.11b radios collected in an industrial environment, showing that packet losses occur in bursts [10]. Woo et al. [11], and Zhao and Govindan [12] have both observed a significant variability in link quality in wireless sensor networks with radios in the 433 MHz band. The former paper points out that the instantaneous packet error probability varies by approximately 30% around its mean. The latter paper, as well as Willig et al. both show that the packet-error stochastic process exhibits significant long-term dependence.


We now define our channel and network models, with an eye toward understanding how time-varying channels affect throughput-optimizing routing metrics.

### III. Model

We assume a time-varying binary symmetric channel where a bit transmitted at time $t$ is misdetected by the intended receiver with probability $P_{B,t}$. We assume that $\{P_{B,t}, t \geq 1\}$ is a stationary random process that is independent of the channel input. In this somewhat unusual (but realistic) model, $P_{B,t}$ represents two things:

1. It is a sample outcome of a random process. Each sample, $P_{B,t}$, takes a value between 0 and 1.
2. It is the probability that an event (bit error) occurs at time $t$.

Note that we will not attempt to model $\{P_{B,t}, t \geq 1\}$. We assume only that it is stationary. The statistics of this process characterizes the channel variability (e.g., due to fading, mobility and multiuser interference).

For simplicity, we assume fixed-sized packets of size $S$ bits. For a packet to be received error-free by the receiving node over a radio link, the sender needs to send all $S$ bits without an error. We assume that in each packet CRC bits accompany data bits to detect the presence of errors. If the receiver detects an error, it drops the packet and does not send a link-layer acknowledgment (ACK) to the sender. The absence of an ACK causes the sender to retransmit the packet. The failure of a threshold number, $M$, of link-layer transmissions causes the link-layer sender to drop the packet, and the packet loss is visible to the higher-layer protocol (which may retransmit the packet end-to-end, as in TCP).

Let us define the discrete time process $\{P_{c,k}, k \geq 1\}$ as

$$P_{c,k} = \prod_{t=k}^{t_k+S-1} (1 - P_{B,t}).$$

where $t_k$ is the starting time for the transmission of the $k$th packet (it may be the initial transmission or a retransmission). Since $\{P_{B,t}, t \geq 1\}$ is stationary, $\{P_{c,k}, k \geq 1\}$ is also a stationary process.

We assume that the channel is “conditionally memoryless”. That is, conditioned on the values of the samples, $P_{B,t_1} = p_1, \ldots, P_{B,t_i} = p_i$, the probability of observing no errors for any of those bits is $\prod_{j=1}^{i}(1 - p_j)$. Thus, $P_{c,k}$ represents the conditional probability that bits $t_k, \ldots, t_k + S - 1$ are all transmitted without an error given $P_{B,t}$ (for $t_k \leq t \leq t_k + S - 1$).
Consequently, the unconditional probability that the corresponding packet contains no errors is $E[P_{c,k}]$, which is constant due to stationarity. Again for simplicity, let us assume that the successive transmissions of the same packet correspond to successive samples of $\{P_{c,k}\}$. For instance, $P_{c,1}$ is associated with the first transmission of the first packet, and $P_{c,2}$ with the second transmission of the same packet given that the first packet has errors.

With the above notation, the ETX (as defined in [1]) of a channel is $(E[P_{c,k}])^{-1}$. This definition implies that $\{P_{c,k}, k \geq 1\}$ is assumed to be an iid process, and hence the number of transmissions is from a geometric distribution.

In Section II-B, we discussed how wireless channels vary over both short (single-packet) and longer time-scales. Thus, the iid assumption is not reasonable and $(E[P_{c,k}])^{-1}$ is not necessarily a good indicator of the quality of a wireless channel. In [17], Koksal et al. showed that the variability in both the long and the short time scale has a significant impact on the expected number of transmissions. For example, given two links, the link with a lower ETX metric may in fact lead to a higher observed loss rate at the transport layer, because good link-layer protocols do not try to retransmit lost packets forever but give up after a threshold number of attempts. When losses occur in bursts, picking the link in the middle of a burst-error situation would be bad even if it had a lower ETX. A routing metric should therefore consider both the mean (ETX) and the variability of the channel.

In the rest of the paper, when we say “a packet” we mean the initial transmission as well as the possible retransmissions until the packet is received error free (or given up on). To refer to a single transmission (or a retransmission) we simply use the term “transmission.”

IV. mETX: MODIFYING ETX TO HANDLE VARIABILITY

Let us call $1/P_{c,k}$ the instantaneous number of transmissions. It signifies the average number of transmissions for correct reception if the probability of an error-free packet remained fixed at a given $P_{c,k} = p_{c,k}$ for every possible retransmission until the packet is received correctly. In this section we study the process $1/P_{c,k}$ and evaluate the expected number of transmissions in the presence of possible dependence between bit errors in successive transmissions. We use this analysis to develop the mETX metric.

Define $\eta_{B,t} = -\log(1 - P_{B,t})$. It can be shown (see the appendix of [17]) by basic algebra that

$$P_{B,t} \leq \eta_{B,t} \leq P_{B,t} + \frac{P_{B,t}^2}{1 - P_{B,t}},$$

for all $t$. Thus, $\eta_{B,t} \approx P_{B,t}$ for reasonably small values of $P_{B,t}$. For instance, if a sample bit-error probability, $p_{B,t}$, is $10^{-3}$, then $\eta_{B,t} = 1.001 \times 10^{-3}$. It may be helpful and more intuitive for the reader to think that the two are replaceable unless mentioned otherwise. Using Eq. (1), we can write

$$\frac{1}{P_{c,k}} = \exp \left( \sum_{t=t_k}^{t_k+S-1} \eta_{B,t} \right).$$

Let $\Sigma_k = \sum_{t=t_k}^{t_k+S-1} \eta_{B,t}$ and let $g_\Sigma(r) = E[\exp(r\Sigma_k)]$ be the moment generating function of $\Sigma_k$. Then,

$$E \left[ \frac{1}{P_{c,k}} \right] = E[\exp(\Sigma_k)] = g_\Sigma(1) = \exp (\Lambda_\Sigma(1)),$$

where $\Lambda_\Sigma(r) = \log g_\Sigma(r)$.

Let $\mu_\Sigma$ and $\sigma_\Sigma^2$ be the mean and the variance of $\Sigma_k$. If we expand $\Lambda_\Sigma(r)$ using the cumulant expansion, we can write (4) as

$$E \left[ \frac{1}{P_{c,k}} \right] = \exp \left( \mu_\Sigma + \frac{1}{2} \sigma_\Sigma^2 + f(\text{higher order terms}) \right),$$

where $f(\text{higher order terms}) = 0$ if $\{\Sigma_k, k \geq 1\}$ were a Gaussian process. For a detailed treatment of the above analysis with variability at different time scales, see [17].

We call define the modified ETX (mETX) as $E[1/P_{c,k}]$, expanded up to the second order cumulant, i.e.,

$$\text{mETX} = \exp \left( \mu_\Sigma + \frac{1}{2} \sigma_\Sigma^2 \right).$$

Note that mETX $\geq$ ETX by Jensen’s inequality. Thus, mETX combines two components, $\mu_\Sigma$ and $\sigma_\Sigma^2$, as representatives of the average and the variability of the error probability respectively. The instantaneous number of transmissions, $1/P_{c,k}$, implicitly assumes that a sample $P_{c,k} = p_{c,k}$ is constant during the entire packet duration unlike the ETX, which assumes $\{P_{c,k}, k \geq 1\}$ is an iid sequence. Hence, to measure $1/P_{c,k}$, we focus on the time scales of order at least a single packet.

Another way of understanding this point is from Eq. (6). The mETX increases with the term $\mu_\Sigma$, which represents the average level of the channel bit error probability over long periods of time. The packet-to-packet variability of $\Sigma_k$ is captured by the $\sigma_\Sigma^2$ term. Therefore, using mETX we can differentiate between channels with different variability at the packet time scale, which is the relevant time scale if the number of transmissions is under consideration. On the other hand, only $\mu_\Sigma$ suffices to calculate the ETX of a channel.

Similar to the ETX metric, mETX is additive over successive links. Therefore, we can use the protocol built for ETX by simply replacing ETX with mETX (see §VII).

V. ENT: EFFECTIVE NUMBER OF TRANSMISSIONS

We now consider the problem of optimizing aggregate throughput, while bounding the packet loss rate visible to higher-layer protocols such as TCP. In this scenario, picking the path that maximizes the link layer throughput may not be sufficient, because it may involve links with high loss rates. Because a good link-layer protocol will give up after a certain threshold number of retransmissions ($M$), ETX and mETX may pick links that violate the loss rate visible to higher layers. The ENT metric meets the desired goal.

Consider the probability that the instantaneous number of transmissions, $1/P_{c,k}$, exceeds $M$. This event is identical to
the threshold crossing for the moving average of the process \( \{ \eta_{B,t}, \ t \geq 0 \} \):

\[
P \left( \frac{1}{P_{c,k}} \geq M \right) = P \left( \sum_{t=k}^{k+S-1} \eta_{B,t} \geq \log M \right)
\approx \exp \left[ -\frac{1}{2} \left( \frac{\log M - \mu_\Sigma}{\sigma_\Sigma^2} \right)^2 \right]. \tag{7}
\]

The derivation is based on a large deviation analysis, as detailed in Theorems 1 and 2 in the appendix.

We used two approximations in the RHS of Eq. (7). The first is the Gaussian approximation for \( \Sigma_k \), and the second is the Chernoff bound.

Unless \( \Sigma_k \) has sufficiently small higher-order cumulants (e.g., skewness, kurtosis) or is already a Gaussian process,\(^3\) the Gaussian approximation is not necessarily accurate. It becomes reasonable if \( (\mu_\Sigma, \sigma_\Sigma^2) \) is such that \( \log M \ll \mu_\Sigma + \sigma_\Sigma^2 = \log \text{mETX} + \frac{2}{\sigma_\Sigma^2} \). For instance, if the packet size \( S \gg 1 \), then the Gaussian approximation is reasonable because both \( \mu_\Sigma \) and \( \sigma_\Sigma^2 \) grow with \( S \).

Eq. (7) is actually an upper bound (the Chernoff bound) for the actual probability. Similar to the Gaussian approximation, the Chernoff bound becomes tighter as \( M \) grows larger and if \( M \) is such that \( \log M \gg \sqrt{2} \sigma_\Sigma \). To summarize, (7) becomes increasingly accurate as \( S \) and \( \log M \) grow large (in a certain way). See the appendix for a more formal treatment.

Let us give an example to illustrate the tightness of the Chernoff bound in a typical scenario. The following is based on Theorem 2. Suppose we would like to check whether (7) gives us the actual probability within \( \pm 10\% \) accuracy. That is, we require that the instantaneous number of transmissions is no less than 0.9\( M \) with a probability identical to the RHS of (7). This will be realized if \( \log M \geq 10 \sqrt{2} \sigma_\Sigma \). Let \( S = 10^4 \) and \( \sigma_{P_{b,t}} \approx \sigma_{\eta_{B,t}} = 10^{-5} \). Since, \( \sigma_\Sigma \leq \sqrt{\sigma_{\eta_{B,t}}} \), it suffices that \( M > 4.1 \) for the required accuracy to be realized.

Noting the above caveats, we shall pursue the development of the ENT with the Gaussian approximation. That is, we will take into consideration only the first two order statistics of \( \Sigma_k \) in evaluating the links that we measure. We believe the Gaussian approximation not only simplifies the algorithm and the measurements that the algorithms are based on, but also captures the variability and the burstiness of the channel error probability sufficiently well at the time scales that are relevant to our problem.

ENT takes into account the probability that the number of transmissions exceed a certain threshold. Let

\[
P_{\text{loss}} = P \left( \frac{1}{P_{c,k}} > M \right), \tag{8}
\]

for any given \( k \). Also let \( \text{mETX} \ll M \) (otherwise \( P_{\text{loss}} \) will be very high). Suppose the application (e.g., TCP) requires that \( P_{\text{loss}} \) to not exceed a given threshold over a given link (TCP performs poorly as loss rates rise). Now, we will express this threshold in such a form that the probabilistic constraint will reduce to a linear constraint involving \( \mu_\Sigma \) and \( \sigma_\Sigma^2 \). For any given threshold probability, there exists a \( \delta > 0 \) such that the value is identical to \( \exp (-\delta (\log M - \mu_\Sigma)) \). Combining (7) with the constraint \( P_{\text{loss}} \leq \exp (-\delta (\log M - \mu_\Sigma)) \), we get

\[
\delta \leq \frac{1}{2} \frac{\log M - \mu_\Sigma}{\sigma_\Sigma^2}.
\]

Thus, a link satisfies (8) if

\[
\mu_\Sigma + 2\delta \sigma_\Sigma^2 \leq \log M. \tag{9}
\]

One way to interpret the condition in (9) is as follows. Suppose the higher layer does not specify any loss probability constraint, i.e., \( \delta = 0 \). The condition turns into a comparison of \( \mu_\Sigma \) (hence the average bit error probability of the channel) with \( M \). Thus, the higher-layer requirement turns into a condition involving average channel parameters only, as is the case with ETX. Now suppose the higher-layer has a loss rate requirement, i.e., \( \delta > 0 \). In that case we need to underbook the resources to meet the loss probability target. The amount of spare \( \mu_\Sigma \) that has to be put aside in order to accommodate single packet time scale fluctuations is \( 2\delta \sigma_\Sigma^2 \). This underbooking allows the packet loss probability target to be met. As expected, this amount is directly related to the variability, \( \sigma_\Sigma^2 \), of the channel and the strictness, \( \delta \), of the loss rate requirement.

The first and second terms on the left side of (9) are the expected value and the scaled version of the variance\(^4\) of the log instantaneous number of transmissions, respectively. The \( \mu_\Sigma \) term represents the impact of slowly varying and static components in the channel, while the \( \sigma_\Sigma^2 \) represents the impact of relatively rapid channel variations (at the packet time scale). We managed to represent the combined impact of the two in (9). In a sense, we projected them onto the same dimension using the higher-layer loss probability, \( P_{\text{loss}} \).

We compare the sum of these two with \( \log M \) maximum number of transmissions before a packet loss manifests at the higher layer (because the maximum number of link-layer transmissions, \( M \) has been exhausted). The unit of the right side is log number of transmissions and so is the unit of the left side. This sum can hence be thought of as the logarithm of the effective number of transmissions (i.e., \( \log \text{ENT} \)) of the link. Note that, if the transmission power for a link is fixed, ENT can also be interpreted as the effective energy for a packet transmitted over the link.

ENT has a structure similar to mETX. The main difference is the extra degree of freedom due to the factor \( 2\delta \). Indeed, the mETX is the ENT evaluated at \( \delta = 1/4 \). Similar to the mETX, a by-product of ENT will be to reduce the packet loss ratio observed by higher-layer protocols, after any link-layer retransmissions are done.

For a wireless link, we define

\[
\alpha(\delta) = \mathbb{E} \left[ \log \frac{1}{P_{c,k}} \right] + 2\delta \text{var} \left( \log \frac{1}{P_{c,k}} \right)
\]

as the log ENT (and hence \( \exp(\alpha(\delta)) \) is the ENT). If a link satisfies \( P_{\text{loss}} \leq \exp(-\delta \rho) \), then the log ENT for that link is between the expected log instantaneous number of

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\(^3\)One has to be careful because \( 0 \leq \Sigma_k \) with probability 1 for all \( k \).

\(^4\)As \( S \) grows, \( \sigma_\Sigma^2 \) approaches index of dispersion of the process \( \eta_{B,t} \).
transmissions and the maximum number, \( M \), of link-layer transmissions before a packet loss is observed at the sender’s transport layer. As a result, one can write the following condition for the link.

\[
\alpha(\delta) \leq \log M \ \Rightarrow \ P_{\text{loss}} \leq \exp(-\delta\rho).
\] (10)

A reader familiar with the notion of effective bandwidth for traffic sources in queueing networks (see e.g., [18]) may realize that the ENT is analogous to the effective bandwidth. The effective bandwidth quantifies the variability of a traffic source and combines it with its mean to determine the contribution of the source to the buffer overflow probability. Analogously, the ENT of a link can be viewed as the effective bandwidth of the number of transmissions and it quantifies the consumed amount of “resource,” which is expressed in terms of a maximum number of transmissions, \( M \) before a loss occurs. Note that a similar second-order approximation for the effective bandwidth to the one that we used in our derivation can be found in [19].

VI. GEOMETRIC INTERPRETATION OF QAR METRICS

To shed some light on the mETX and ENT metrics and their relation to other quality-aware metrics, we now present a geometric interpretation of QAR metrics.

Let us represent a wireless link by two parameters, \( \mu_\Sigma \) and \( \sigma_\Sigma \). Each link corresponds to a point in the coordinate space \((\sigma_\Sigma, \mu_\Sigma)\) as illustrated in Fig. 2(a). In our graphs we use \( \mu_\Sigma - \log M \) instead of \( \mu_\Sigma \) as the y-axis. This only introduces a linear shift and it simplifies our discussion. In this space, the point with the lowest ordinate value is the one that minimizes the ETX. Such links will be preferred by routing algorithms that employ \( \mu_\Sigma \) as the link cost metric (e.g., ETX).

The set of points that satisfy \( P_{\text{loss}} = \exp(-\delta(\mu_\Sigma - \log M)) \) are on the parabola \( \alpha(\delta) = \log M \) as shown in Fig. 2(b). Thus, the points outside the shaded region fail to satisfy \( P_{\text{loss}} \leq \exp(-\delta(\mu_\Sigma - \log M)) \). The shaded region can therefore be regarded as a feasible region. Notice that for \( \delta = 0 \) (i.e., no loss-rate requirement) the feasible region is the entire fourth quadrant. The region shrinks as \( \delta \) is increased since the boundary of the region becomes more concave. Hence, a smaller number of links become feasible.

Suppose we want the routing algorithm to select the links that not only minimize the expected number of transmissions, but also keep the loss probability smaller than \( \exp(-\delta(\mu_\Sigma - \log M)) \). Then, the algorithm should pick the link with the smallest ordinate value among the points in the feasible region. We outline such a path selection algorithm in Section VII.

Similarly, the set of links with a constant mETX constitute a parabola in the coordinate space \((\sigma_\Sigma, \mu_\Sigma)\). Indeed, the set of points with mETX equal to \( c \) is on the parabola specified by

\[
\mu_\Sigma + \frac{1}{2} \sigma_\Sigma^2 = c.
\]

These parabolas can also be viewed as the boundaries for a feasible region, where the feasible links are those with mETX less than some given \( c \) value. Identical mETX curves are illustrated in Fig. 2(c). As \( c \) is reduced, the boundary moves farther away from the x-axis and the set of points with smaller mETX shrink.

For any given point, the slope of the line connecting the origin to that point is \( (\mu_\Sigma - \log M)/\sigma_\Sigma \). Combining this with Eq. (7), points with smaller slopes, i.e., points with larger \( (\mu_\Sigma - \log M)/\sigma_\Sigma \) have lower loss probabilities. For instance, in Fig. 2(d), channel \( l \) has a higher loss probability than channel \( l' \). Thus, two links with an identical ENT for a given \( \delta \) may have different loss probabilities. If the objective is to minimize the probability of loss, then the path selection algorithm should choose points with large \( (\mu_\Sigma - \log M)/\sigma_\Sigma \) ratios.

Finally, consider the vertical distance, \( D(l) \), between any admissible point, \( l : (\sigma_\Sigma, \mu_\Sigma) \) and the boundary of some feasible region of links. As illustrated in Fig. 2(e),

\[
D(l) = -(\mu_\Sigma - \log M) - 2\sigma_\Sigma^2 \\
= \log M - \alpha(l)(\delta)
\] (11)

Hence, the link that maximizes the vertical distance to the boundary of the feasible region is the one that minimizes the ENT. This means that, given an increase in the expected number of transmissions, the link with a small ENT is more likely to remain in the admissible region. Thus, if the objective is robustness with respect to the uncertainty in the measured parameters and to changes in the expected number of transmissions, the routing algorithm should choose points with smaller ENT.

VII. PATH SELECTION WITH mETX AND ENT

We now outline two path selection algorithms based on mETX and ENT. In terms of disseminating routing information, these algorithms are similar to the ETX algorithm (e.g., they could use protocols like DSR, AODV, DSDV, etc. [20]). The only difference from the ETX algorithm is that the parameter \( \sigma_\Sigma \) is propagated in addition to \( \mu_\Sigma \).

A. Estimating the Channel Parameters

To estimate the channel parameters, we measure the channel using link-layer probe packets. This scheme is similar to the one described in [1] to estimate the ETX, except the data we record is at the bit level for each probe, rather than at the packet level as in [1]. A probe packet contains a known sequence of bits. Each node periodically broadcasts a probe packet; the broadcast nature of the wireless medium allows multiple receivers to measure the quality of the channel and themselves using the same sender probe. Moreover, the data packets\(^5\) can also be used to estimate channel quality (passive channel probing).

The parameters \( \mu_\Sigma \) and \( \sigma_\Sigma^2 \) of a channel are estimated by considering the locations of erred bits in each probe packet. As in [1], each node sends 10 probe packets to calculate a loss rate sample, and this information is passed to exponentially weighted moving average (EWMA) filter whose weight is halved every 5 samples.

\(^5\)Once a packet is correctly decoded, the number of bit errors in all the previous transmissions can be evaluated.
B. Routing Algorithms

Next, we give the routing algorithms. The associated routing metric for each link is calculated using \( \mu_\Sigma \) and \( \sigma_\Sigma^2 \).

**Algorithm 1**: Compute the mETX and assign it to each link as the cost for routing over that link. Between any pair of nodes, use the path that minimizes the total cost.

**Algorithm 2**: For each link, compute its \( \log \text{ENT} \). Compare against \( \log M \). Assign a cost of \( \infty \) to the links that have \( \log \text{ENT} > \log M \) and assign a cost of ETX to the others. Between any pair of nodes use the path that minimizes the total cost.

This algorithm focuses only on the feasible links, i.e., the ones that satisfy the application loss requirement, \( P_{\text{loss}} \). It picks those with the minimum ETX among those. This involves only a minimal modification to ETX (given the link parameters).

Note that since the ENT is a function of the higher-layer parameter \( \delta \), an optimal path between two nodes may differ from one application (or transport layer) to another in Algorithm 2.

VIII. MEASUREMENTS AND ILLUSTRATION

This section presents experimental and simulation results to illustrate the performance of mETX and ENT in practice. The section is divided into two parts: link-layer measurements and network-level simulations. Our link measurements are taken from Roofnet, a community mesh network composed of 802.11b-based nodes spread over a 4 square kilometer region in Cambridge, MA. Each node has an omni-directional antenna [9]. Using these measurements we simulate different sized mesh networks with links similar to those in Roofnet.

A. Link Level Measurements and Observations

In the previous section, we showed that each link can be represented with a point in the \( (\sigma_\Sigma, \mu_\Sigma) \) space and described how to estimate these two values. Now, we first estimate these parameters for the Roofnet links and study how they are spread in the \( (\sigma_\Sigma, \mu_\Sigma) \) space. A uniform spread of points in this space would suggest that mETX and ENT would be useful in practice, whereas a pattern where \( \mu_\Sigma \) increases with increasing \( \sigma_\Sigma \) would suggest that there is no additional information to be gained from including the variance in the quality estimator.

Fig. 3 shows the results of measurements taken from the Roofnet testbed at a transmission rate of 11 Mbps. The parameters are estimated as each node in turn sends 802.11 broadcast packets at constant rate for approximately 20 seconds, while the rest of the nodes listen passively. In this picture, there are 57 links that belong to distinct pairs of 12 different nodes. It shows that the radio links are spread quite uniformly in the space, with little correlation between \( \sigma_\Sigma \) and \( \mu_\Sigma \) of a link for
1.21. Between 223 and 740, the minimum ETX path is the direct link since the ETX is lower bounded by 1 and any link pair would have an ETX of at least 2. However, as shown in the figure, the direct link is not feasible. Next, consider using node 652 as a relay. The two hop path, has links with an ETX of 1.17 and 1.04 respectively. The total ETX of this path is 2.21, which is 1 larger than that of the direct link.

Actually, we did not need to do an extensive search over different paths to come up with such an example, where ETX picks a suboptimal path. Indeed, in our analysis 194 link pairs, \((l_1, l_2)\), have the property that ETX\((l_1) < ETX(l_2)\) and \(\sigma_{\Sigma}^{(l_1)} > \sigma_{\Sigma}^{(l_2)}\). This corresponds to 12.2% of all 1596 link pairs. Among these 194 pairs, 80 of them further satisfy mETX\((l_1) > mETX(l_2)\). Moreover, in Fig. 4(a), 4(b) and 4(c), we illustrate \(P_{loss}\) (as given in Eq. (7)) versus \(\mu_{\Sigma}, \sigma_{\Sigma}\) and mETX respectively. While there is no obvious trend between \(P_{loss}\) and \(\mu_{\Sigma}\) (and hence the ETX), \(P_{loss}\) almost always increases with \(\sigma_{\Sigma}\). Indeed, the correlation coefficient between \(P_{loss}\) and \(\mu_{\Sigma}\) is 0.59, whereas this value is 0.85 between \(P_{loss}\) and \(\sigma_{\Sigma}\). Also note that the correlation coefficient between \(P_{loss}\) and \(\log \text{mETX}\) is 0.94, which implies that mETX combines \(\mu_{\Sigma}\) and \(\sigma_{\Sigma}\) to measure the loss rate much more accurately than it could be measured with ETX alone.

Therefore, if the loss rate is the parameter of interest, one should definitely consider the variability as well as the channel average in a routing cost metric, as we do in the mETX and the ENT. Consequently, we expect a significant change in the end to end optimal paths with our metrics compared to the ETX. We show this in the following section.

B. Network-Level Simulations

Now that we have a better understanding of how our metrics and loss probabilities behave on a single link, we are ready to simulate the network level behavior of the routing algorithms we present. Our simulations shall be “honest” in the sense that we will use actual Roofnet link measurements in the construction of our networks.

We use the following randomized procedure to construct the network topology. We pick a random number of nodes between 8 and 25. Then, we assign one of the 57 Roofnet links between a randomly selected pair of nodes (the link between the randomly selected pair of nodes has the same \(\mu_{\Sigma}\) and \(\sigma_{\Sigma}^2\) as the assigned Roofnet link). We make the assumption of symmetric links, i.e., the downlink and the uplink between any given node pair is identical.

Our construction leads to both sparse and dense networks in terms of connectivity. First, we keep the total number of links constant at 57 as we keep increasing the number of nodes. For a network with 12 or more nodes, there are at least 66 node pairs. For such a network, since there are fewer links to assign than node pairs, some pairs will remain unassigned. We assume such node pairs to be out of the transmission range of each other, i.e., no direct link exists between the two. The outcome is a sparse (possibly partitioned) network. Second, we expand the number of potential links, again using the same Roofnet measurements. Hence, every node pair is assigned a link (can be one of the poor quality links, though), and the resulting structure is a dense (and connected) network. Note that the actual link measurements we take from the Roofnet at 11 Mbps belong to a total of 12 nodes. We also simulate the actual Roofnet among other randomly formed 12 node networks with randomly constructed topologies.

Once the topology of the network is formed, we construct the routing tables. Each entry of a routing table will give the
optimal path between a pair of nodes based on the ETX, the mETX or the ENT metrics. For the latter two metrics, we use algorithms 1 and 2 respectively for the construction of the routing tables. Note that we calculate the optimal paths centrally, since our purpose is to compare these metrics rather than to analyze the dynamics of the distributed routing protocols.

Once the routing tables are constructed, we calculate the overall higher layer loss rate, $P_{loss}$, between each pair of nodes. If the path consists of multiple links, the overall loss rate is calculated using the assumption that the successive links over a path have independent losses. For instance if $l_1$ and $l_2$ connect two nodes, the overall loss rate over the path is $P_{loss} = 1 - (1 - p_{l_1})(1 - p_{l_2})$ where $p_{l_i}$ is the loss rate of link $l_i$. We define the network loss rate as the loss rate averaged over each path between every possible node pair. Note that the loss rate between a pair of nodes is taken to be 1 if it is not possible to connect the pair with existing links of $P_{loss} < 1$.

In Fig. 5(a), the network loss rate with the ETX and the mETX metrics are illustrated as a function of the number of nodes. Each point is calculated by averaging over 25 randomly generated topologies of a given number of nodes. As expected, the difference between the network loss rates of the mETX and the ETX based algorithms is in favor of the former. This difference may be as high as 7%, especially when the number of nodes is small.

A high packet loss rate (after possible retransmissions) degrades end-to-end TCP throughput. In the steady state, the average TCP throughput for a log-running transfer is roughly proportional to $1/\sqrt{P_{loss}}$. Thus, a 7% reduction in the loss rate from 0.21 to 0.14 corresponds to an increase of 22% in the TCP throughput supported within the entire network. The improvement is much more dramatic in the dense networks. The increase in the TCP throughput is as high as 60% here. Also note that based on the simulation with the actual Roofnet topology, the network loss probability decreases from 19% to 12%, which corresponds to a 30% increase in the average TCP throughput with mETX.

The increase in the loss rate with increasing number of nodes is plausible in sparse networks, since the total number of links assigned is fixed at 57. A larger number of nodes are unable to connect over a direct link as the number of nodes increases. This leads to an increase in the number of nodes that cannot be connected even over multiple hops. We take the loss rate between such pairs to be 1. Similarly, the network loss rate decreases with increasing number of nodes in dense networks, because for every pair of nodes, the potential number of paths increases with increasing number of nodes.

In every random topology generated, we observe that the maximum number of hops between any given pair of nodes is 3 and rarely this goes up to 4 with both the mETX and the ETX routing metrics. We plot the fraction of the paths that differ for the ETX and the mETX based algorithms in Fig. 5(b). The fraction goes from 35% with 12 nodes to 47% with 25 nodes for sparse networks and around 30% for dense networks. Thus the ETX and the mETX based algorithms come up with the same paths for about no more than 70% of the node pairs in our simulations.

Next we analyze Algorithm 2. This time we only focus on the dense network scenario. The set of feasible links are those with $\alpha(\delta) < M = 16$. Between each pair of nodes, the algorithm picks a path consisting of feasible links that minimizes the total ETX. With the same procedure of topology generation and random assignment of Roofnet links, we plotted the network loss rate as a function of number of nodes for the second algorithm with ENT evaluated using a range of application parameter values between $\delta = 1$ and $\delta = 2.5$ in Fig. 5(c). We also plotted the network loss rate with pure ETX (which corresponds to the case where $\delta = 0$) on the same graph. Similar to the previous simulations, we use the average of 25 different random topologies for every given number of nodes.

We observe some interesting trends here. First, we can control the “quality of service” (in terms of observed loss rates) provided to the applications by just adjusting a single parameter. Indeed, the space parameter $\delta$ acts as a knob to control the performance and the loss rate can be reduced by an amount of 7-8% (and hence the TCP throughput can be
increased by up to 55%) by adjusting the value of the space parameter. There is a catch, though. The loss rate does not decrease monotonically as we keep increasing $\delta$. It can be seen in Fig. 5(c) that as we move $\delta$ from 2 to 2.5, the loss rate increases. The reason for this transition is that we start to eliminate too many links beyond a certain critical threshold. This threshold is somewhere between $\delta = 2$ and $\delta = 2.5$ in Roofnet for this data-set. Consequently, even many “decent” links are gone and no feasible paths remain between some node pairs.

IX. CONCLUSION

In this paper, we addressed the problem of selecting good paths in wireless mesh networks. We developed mETX and ENT, two new quality-aware routing metrics for such networks. These metrics are derived using a time-varying binary symmetric channel model and they combine the average and the standard of observed channel loss rates. The metrics are designed to maximize the network throughput and at the same time reduce the loss rates visible to higher layer protocols like TCP, after the link layer has attempted to deliver the packet using retransmissions. Our analysis shows how a wide range of QAR metrics fit into a unified framework. Network designers can choose from a number of different metrics using this framework.

We evaluated mETX and ENT using measurements from Roofnet, a real-world wireless mesh network, and illustrated how these metrics give us the flexibility of controlling the loss rates of the optimal paths found by the routing algorithms. Empirical observations of this network suggest that mETX and ENT could achieve a 50% reduction in the average packet loss rate compared to ETX. We believe that the insights developed in this paper will prove useful to the designers of wireless mesh and sensor networks in coping with radio channel variations.

In this paper, we abstracted all the variations of the channel into the stochastic process, $P_{B,T}$. An extension of this work is to build our metric based on physical layer parameters such as the SNR. We believe that this extension could shed further light on the impact of the physical layer on optimal routing decisions. It could simplify channel estimation as well. Another extension is to study the impact of adaptive coding and power control on routing.

ACKNOWLEDGMENTS

We thank Dan Aguayo and the other members of the Roofnet project [21] for sharing their data. We also thank Douglas De Couto, Stephen Hanly, Robert Morris, Emre Telatar, Patrick Thiran, and Greg Wornell for useful discussions. This work was supported in part by NSF Grant CNS-0520032.

REFERENCES

APPENDIX

Derivation of the Probability of Packet Loss

In this section, we derive the probability of packet loss, given in (7). Recall that \( \eta_{B,t} = -\log(1 - P_{B,t}) \), \( \Sigma = \sum_{t=t_k}^{t_k+S-1} \eta_{B,t} \), \( \mu_S = \mathbb{E}[\Sigma] \) and \( \sigma^2_S = \text{var}(\Sigma) \). We assume that

\[
\Lambda_S(r) = \frac{1}{S} \log \mathbb{E} \left[ \exp \left( r \sum_{t=1}^{S} \eta_{B,t} \right) \right]
\]

exists for some \( r_- < r < r_+ \) and is differentiable for \( 0 \leq r \leq r_+ \) (Note that this quantity is the normalized log moment generating function of \( \Sigma \)). Let \( \rho = \log M - \mu_S \). The following theorem gives an upper bound for the packet loss probability.

**Theorem 1:**

\[
\log P \left( \frac{1}{P_{c,k}} \geq M \right) \leq -S \Lambda^*_S \left( \frac{\rho}{S} \right),
\]

where

\[
\Lambda^*_S \left( \frac{\rho}{S} \right) = \sup_{r \geq 0} \left[ \frac{\rho}{S} r - \left( \Lambda_S(r) - \mu_S \right) \right].
\]

The proof is simply a consequence of the Chernoff bound. Note that

\[
\log P \left( \frac{1}{P_{c,k}} \geq M \right) = \log P \left( \sum_{t=t_k}^{t_k+S-1} \eta_{B,t} \geq \log M \right) = \log P \left( \sum_{t=t_k}^{t_k+S-1} \eta_{B,t} - \mu_S \geq \rho \right).
\]

Inequality (12) is the Chernoff bound applied to the probability in (14). The bound is asymptotically tight with \( \rho \). The optimization procedure in Eq. (13) is illustrated graphically in Fig. (6). The abscissa of the point where \( \Lambda_S(r) - r \mu_S / S \)

has a derivative of \( \rho / S \) is \( r_m \). The tangent line at \( r_m \) cuts the x-axis at \( r_o \). Thus,

\[
r_o = \frac{S}{\rho} \Lambda^*_S \left( \frac{\rho}{S} \right) = \frac{S}{\rho} \left( \rho r_m - \left( \Lambda_S(r_m) - \mu_S \right) \right).
\]

This implies,

\[
P \left( \sum_{t=t_k}^{t_k+S-1} \eta_{B,t} \geq \log M \right) \approx \exp(-r_o \rho).
\]

If we approximate the log moment generating function with the first two terms of the Taylor series expansion, we get

\[
\Lambda_S(r) \approx \rho \mu_S + \frac{1}{2S} \sigma^2_S r^2,
\]

since \( \Lambda'_S(0) = \mu_S / S \) and \( \Lambda''_S(0) = \sigma^2_S / 2S \). The ‘\( \approx \)’ sign can be replaced with ‘=’ if \( \eta_{B,t} \), \( t \geq 1 \) is a Gaussian process. Hence, this approximation is also called the Gaussian approximation. The Gaussian approximation for the log moment generating function, \( \Lambda_S(r) \), is reasonable for \( r \ll 1 \). Suppose \( r_m \ll 1 \). Using (16), \( r_m \) can be evaluated as

\[
r_m = \frac{\rho}{\sigma^2_S}.
\]

Hence \( r_o = r_m / 2 \). Note that, generally \( \sigma^2_S \gg 1 \) due to the small decay in the autocovariance of \( P_{B,t} \) even in very long time scales. Then, the relevant \( r \) values are reasonably small \( (r_m \ll 1) \), i.e., \( \log M < \mu_S + \sigma^2_S / 2 \log(1 + \frac{1}{2} \sigma^2_S) \). Let us rewrite Eq. (15):

\[
P \left( \sum_{t=t_k}^{t_k+S-1} \eta_{B,t} \geq \log M \right) \leq \exp \left[ -\frac{1}{2} \left( \frac{\log M - \mu_S}{\sigma_S} \right)^2 \right].
\]

Now, we discuss the tightness of the upper bound given in (12). For this purpose, we derive a lower bound for \( P(1/P_{c,k} \geq M) \).

**Theorem 2:** Let \( \{1/P_{c,k}, k \geq 1\} \) be a discrete lognormal process. For any \( \epsilon > \sigma_S \sqrt{2} / S \),

\[
\log P \left( \frac{1}{P_{c,k}} \geq M \exp(-\epsilon c) \right) > -S \left[ \Lambda^*_S \left( \frac{\rho}{S} \right) + \epsilon + \frac{\log 2}{S} \right].
\]
The proof involves the use of tilted random variables (i.e., change of measure). We skip the details due to the space limitations, but similar derivations can be found in textbooks on large deviations theory (e.g., [22], Exercise 7.9 or [23]).

To make the (12) as tight as possible, we should have both sides of the inequality (18) as close to their counterpart in (12) as possible. On the right side of (18), the term \( \log \frac{2}{S} \) shrinks as \( S \) grows, and thus, \( S \gg 1 \). Also, to make the lower bound tight, we are inclined to pick \( \epsilon \ll \Lambda_{\psi}(p/S) \). Thus, \( \epsilon \ll \log M/S \). Indeed, on the left side of (18), we should have \( S \epsilon \ll \log M \), which also implies \( \epsilon \ll \log M/S \). Combining this with the initial constraint given in the theorem, i.e., \( \epsilon \gg \sigma_{\psi}\sqrt{2}/S \), we should have \( \log M \gg \sigma_{\psi}\sqrt{2} \). To summarize, to make the Chernoff bound tight, both \( \log M \) and \( S \) should be sufficiently large.