Rate Performance Objectives of Multi-hop Wireless Networks

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Outline

I. Introduction and problem statement
II. Model of ad-hoc network
III. Our findings
IV. Conclusions
I. Introduction

• Goal: design MAC and routing protocol for given network technology.

**Q: What performance objective to use?**

• Performance objectives in multi-hop wireless networks:
  – Rate based objectives (802.11, UWB, CDMA)
  – Energy based objectives (sensor networks)
  – Combined

• We focus on rate-based objectives
Rate-based Performance Objectives

- **Total capacity**: maximize sum of rates of all flows
- **Max-min fairness**: a rate of a flow cannot be increased at the expense of a flow with an already smaller rate.
- **Proportional fairness**: maximize sum of logs of rates of all flows.
- **Transport rate** of a flow = rate * distance

All above metrics applicable to transport capacities

- We can also define metrics corresponding to these objectives, when evaluating performance rather than designing network.
Efficiency and Fairness

- Antagonism between efficiency and fairness
- Maximizing total capacity is unfair (like in wired networks)
- Max-min fairness is inefficient (unlike wired networks)

Q: Given a network technology, what design objective to use to make a compromise between efficiency and fairness?
Performance Indices

• Q: How to quantify efficiency and fairness?

• *Efficiency index of rate allocation* \( f \): \( \sum f_i / \sum f^*_i \)
  where \( f^* \) is rate allocation that maximizes total capacity.

• *Fairness index of rate allocation* \( f \): \( \cos^2(\alpha) \)
  where \( \alpha \) is angle between \( f \) and max-min fair allocation \( f_{mmf} \)
  when MMF rates are equal, this coincides with Jain fairness index.
II. Model of Ad-hoc Wireless Network

• Physical model properties
• MAC protocol
• Routing protocol and traffic flows
• Power control
Physical Model Properties

- Point-to-point links: no broadcast, relay channels, multi-user detection
- Constant and positive attenuation $h_{ij}$ between any two points $i,j$
- Interference allowed, no collisions.
- Signal-to-noise ratio at the receiver of link $l$: ratio of received power over white noise plus interference of other transmitters.
- Rate $r$(SNR) is strictly increasing function.

![Graphs](Gaussian channel, Ultra-wide band, Stair function)
MAC Protocol

• **Schedule** consists of several slots, each of length $a_n$. In each slot, nodes have different **power allocations** $p_n$.

\[
\alpha_1, p_1 \quad \alpha_2, p_2 \quad \alpha_3, p_3 \quad \alpha_4, p_4 \quad \ldots
\]

• In each slot, node achieves rate $x_n$ as a function of SNR and corresponding coding.

• **Long term average rate** is average rate over all slots

\[
\bar{x} = \sum_{n} \alpha_n x_n
\]

• We assume ideal control plane – no protocol overhead
Routing Protocol and Traffic Flows

- Traffic demand is described by end-to-end flows.
- Each flow is unicast or multicast.
- Each flow is mapped to one path (single-path routing) or more paths (multi-path routing).
- Mathematical formulation of constraints on average rates:

\[ f = F_y, \ x = R_y \]

\[ F_{f,p} = 1 \text{ if path } p \text{ belongs to flow } f, \text{ else } 0 \]
\[ R_{p,l} = 1 \text{ if path } p \text{ uses link } l, \text{ else } 0 \]
Power Constraint

- **Peak power constraint**: maximum power of a symbol in a codebook. Integrated in model through rate function.
- **Transmission power constraint** $P_{\text{MAX}}$: average power of transmission in given slot. Corresponds to average power of codebook used.
- **Long term average transmission power constraint** $P_{\text{MAX}}^{\text{avg}}$: average power dissipated over the schedule. It corresponds to battery lifetime:

$$T_{\text{lifetime}} = \frac{E_{\text{battery}}}{(P_{\text{MAX}}^{\text{avg}} u)}$$

$u$ - fraction of time node has data to send
Optimization Problem

• **Input constraints** (due to technology and user preferences): transmission power constraint, rate function, attenuation

• Given network topology and traffic matrix, we have set of feasible rates and set of feasible transport rates.

**Q:** for each performance objective, find optimal end-to-end rates on given feasible rates and feasible transport rates set.

• Non-convex optimization – heuristic needed sometimes.
III. Finding 1: Max-min Fairness is inefficient

- **Theorem:** Max-min fair rate allocation on arbitrary network, without battery lifetime constraint, has all rates equal.

- **Theorem:** Max-min fair transport rate allocation have all transport rates equal.
Result on Max-Min Fair Allocation is due to Solidarity Property

- *Solidarity property* – a set has solidarity property if one can always trade value of one coordinate for other coordinate.
- MMF allocation on set with solidarity has all coordinates equal.
- Not all convex sets have solidarity property.
- Feasible set of rates of wireless network has solidarity property; Feasible set of transport rates also has solidarity property.

Example **with** solidarity property: Feasible set of wireless network

Example **without** solidarity property: Feasible set of wired network
Application to 802.11 Network

- All nodes have equal probability to gain access to channel.
- All nodes have packets of equal sizes: slower nodes take more time to send packet.
- System is essentially max-min fair.
- Conclusion: All nodes will have the same average rate, regardless of coding used.

Phenomenon is not due to physical layer choice, but due to choice of design objective.
III. Finding 2: Maximizing Total (Transport) Capacity is Grossly Inefficient

• **Theorem:** Asymptotic results on maximizing (transport) capacity
  
  – when power constraint $P^{\text{MAX}}$ goes to infinity, only the most efficient flows will have positive rate; the rates of other flows will be zero.
  
  – The same hold for maximizing transport rates – transport rates and rates of inefficient flows will be zero.
III. Finding 3: Proportional Fairness is Good Compromise

- Total capacity, transport, rate
- Proportional, transport
- Proportional, rate
- MMF, rate
- MMF, transport
- Efficiency index (P_{MAX}/N [dB])
- Fairness index (P_{MAX}/N [dB])
IV. Conclusions

• We analyzed three rate-based performance objectives: max. total capacity, max-min fairness, proportional fairness.

• We defined a general model of wireless network, that incorporates most of the existing networks.

• Our findings on the general model:
  – *Total capacity* is unfair metric, especially for large power constraints; longer and inefficient flows get small or zero rate.
  – *Max-min fairness* is inefficient metric. Under no battery lifetime constraints, all flows get the same rate, that is the rate of the most inefficient flows.
  – *Proportional fairness* maintains fairness while increasing efficiency. It is robust to changes in power constraints. It is the optimal performance objective.
Future Work

• Incorporate power into the metric, rather than in constraint.
• Inspect influence of random fading on the results.